Module-05

→ Quantifying Uncertainity:-

1) Certainity :- (No Belief | No guess) No probability).

Agent must solve the problem by Rule (or) Formula and in Certain condition solution can be identified using definite condition.

2) Uncertainity :- (Belief | Guess | Probability)

Agent must solve the problem by logic and in certain condition Solution can be identified using belief.

★ → Acting under Uncertainity:-

* For 20 kms we can have travelling time as 30 minutes designated as A_{30}.

* But we have to consider the following Uncertainity
- Car breaks down
- No fuel
- Tire blow up
- Accident on the bottleneck
- Heavy rain and road block
- Politicians convoy....

* This problem has no exact solution. Hence we can prefer
- Start 30 minutes before and reach airport : A_{30}
- Start 90 minutes before and reach airport : A_{90}
- Start 10 Hours before and reach airport : A_{600}
- Start 24 Hours before and reach airport : A_{1440}

So now, to decide what will be the best option to decide we make use of Rational Decision (Sensible decision)

* Rational Decision is doing right thing.
* Decision can be taken in two ways based on probability (possibility) and Utility (usefull).

→ Taking decision based on probability :-

* If we start 30 Minutes before and reach airport, A_{30} then possibility (probability) of catching flight is less due to various conditions.

1) Probability to catch flight for A_{30} is 60% = 0.6.
2) Probability to catch flight for A_{90} is 90% = 0.9
3) Probability to catch flight for A_{600} is 95% = 0.95
4) Probability to catch flight for A_{1440} is 99.9% = 0.999

* Probability theory says, A_{1440} is best because more chance of catching the flight.

→ Taking decision based on utility quality of being usefull :-

* If we start 30 minutes before & reach airport, A_{30} then time utility is best less waiting time.

1) Utility for A_{30} is very less no waiting time
2) Utility for A_{90} atleast 20-30 minutes roaming in airport
3) Utility for A_{600} nearly 8-9 hours we have spend time (2 meals cost)
4) Utility for A_{1440} almost 24 hours (3 meals / restroom / sleeping).

→ Conclusion :-

Rational Decision is doing right thing. Now we must take decision with respect to both Probability and Utility theory.

i.e. $\boxed{\text{Decision Theory = Probability Theory + Utility Theory}}$

∴, So, final decision is A_{90} with minimal waiting we can catch flight. such decision is Acting Under Uncertainity.

Uncertainity

Let us have dental problem for uncertain reasoning. A patient went to a dentist and say toothache It may be due to cavity in the tooth.

The Rule:-

Toothache $\Rightarrow$ Cavity

Above rule may not be true, because toothache may be due to gum problem or abscess.

Rule change:-

Toothache $\Rightarrow$ Cavity $\vee$ Gum problem $\vee$ Abscess.

Above rule may not be true, because many disease exist.

Now turn the rule.

Causal rule:-

Cavity $\Rightarrow$ Toothache

Again above rule may go wrong; that is not all cavity may give (toothache) pain. {Young people may not get pain}

* Identify all possible reasons for a cavity to give pain {But Fails}

Reason:-

1) Laziness (Too much work for identification)
2) Theoretical Ignorance (Medical science has no complete theory)
3) Practical Ignorance (Performing All test not feasible CT, MRI)

So cavity : Toothache relation is not a simple one. They leads to Uncertainity.

Similarly Medical domain, Judicial, Business, stock Market, Automobile, repair, Agriculture---gives Uncertainity which leads to degree of belief (probability).

Basic Probability Notation :-

* **Probability Theory.** is defined as a mathematics that gives numerical value of Degree of belief. How likely an event occurs. It helps to deal problem with uncertainity.

① **Play one dice** :- All possible sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$

Possibility to get number TWO : $\omega = \{2\}$ is 1 out of 6 throw given as $P(\omega) = 1/6 = 0.16$

Probability is always b/w 0 & 1 : $0 \leq P(\omega) \leq 1$

Sum of probability is always 1 : $\sum_{\omega \in \Omega} P(\omega) = 1$

② **Play two dice** :- All possible sample space is $\Omega = \{\{1,1\}, (1,2), (1,3) --- -(6,5), (6,6)\}$ 36 Possibility.

Possibility to get same number in both dice (Double) :

$\omega = \{(1,1), (2,2), (3,3), (4,4), (5,5) (6,6)\}$ is 6 out of 36 throw given as $P(\omega) = 6/36 = 0.16$

③ Probability to get sum ELEVEN in both dice (sum 11) : $\omega\{(5,6), (6,5)\}$ is 2 out of 36 throw given as $P(\omega) = 2/36 = 0.055 = 5.5\%$

(Unconditional Probability)

④ Probability to get sum ELEVEN in both dice (sum_{11}) with first dice as 6 (sum_{11} given $Dice_1 = 6$) $\omega = \{(6,5)\}$ is 1 out of 36 throw given as $P(\omega) = 1/36 = 0.027 = 2.7\%$

$P(sum_{11} \mid Dice_1 = 6) = 0.027$.

(Condition Probability)

→ **Probability** :- Problem with uncertain reasoning can be deal with Degree of belief by Probability Theory.

Ex :- Toothache ⇒ cavity.

* Patient went to Dentist for General Checkup Doctor identified as patient having cavity as 20% then P(cavity) = 0.2.

* Patient took appointment with dentist due to tooth ache Now patient having cavity given condition toothache as 60%. then P(cavity | Toothache) = 0.6

* Toothache is true then cavity probability is 60%

* P(cavity | Toothache) = 0.6 → Conditional Probability.

· Conditional probability can be defined in terms of Unconditional probability.

$$P(\text{cavity} \mid \text{Toothache}) = \frac{P(\text{cavity} \wedge \text{Toothache})}{P(\text{Toothache})}$$

Provided P(Toothache) > 0 (must exist)

∴, In above equation LHS is conditional probability and RHS is unconditional probability.

Consider,

$$P(\text{cavity} \mid \text{Toothache}) = \frac{P(\text{cavity} \wedge \text{Toothache})}{P(\text{Toothache})}$$

Product rule :-

$$P(\text{cavity} \wedge \text{Toothache}) = P(\text{cavity} \mid \text{Toothache}) \times P(\text{Toothache})$$

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

Product rule :- $P(a \wedge b) = P(a \mid b) . P(b)$

For a AND b to be true then we need b to be true also for given b, a to be true.

→ **Probility Assertions (statement of Fact):-**

i) **Domain** :- The set of possibility values it can take.

Ex01:- Age Domain : {Kid, Teen, Adult}

* Teen went to dentist for checkup. she has No tooth ache but she has cavity.

$P(\text{cavity} \mid \neg \text{toothache} \wedge \text{Teen}) = 0.1$ (conditional probability)

Ex02:- Weather Domain : {Sunny, Rain, cloudy, Snow}

Delhi weather condition in March month is,

$P(\text{weather} = \text{sunny}) = 0.6$

$P(\text{weather} = \text{rain}) = 0.1$

$P(\text{weather} = \text{cloudy}) = 0.29$

$P(\text{weather} = \text{snow}) = 0.01$

Probability of all possible variable must be 1.

* **Probability Distribution:-** $\mathbf{P}(\text{weather}) = (0.6, 0.1, 0.29, 0.01)$
↓
Bold 'P' represents P.D

* **Joint Probability Distribution :-**

We combine probability of different variables like Cavity and Weather.

Ex:- In a sunny day patient went to dentist and identified for cavity (C)

$P(W = \text{sunny} \wedge C = \text{True}) = P(\text{sunny} \mid C)\, P(C)$ → Product Rule

$$\boxed{P(W \wedge C) = P(W \mid C)\, P(C)}$$

In a sunny day patient went to dentist and identified for No cavity (C).

$P(W = \text{sunny} \wedge C = \text{False}) = P(\text{sunny} \mid \neg C)\, P(\neg C)$ → Product rule.

$$\boxed{P(W \wedge \neg C) = P(W \mid \neg C)\, P(\neg C)}$$

Same for 4 weather results give 8 general equations.

*→ We combine probability of different variables like Cavity, Toothache and weather.

* Probability Distribution for above 3 possibilities give

  $2 \times 2 \times 4 = 16$ possible World.

* A Possible World is defined to be an assignment of values to all of the random variables under consideration.

* Full Joint Probability Distribution is given by,

  P (cavity, Toothache, Weather)

* It is representation probability of all 16 possible World.

→ **Probability Inference :-**

Probabilistic Inference is information from Full Joint Probability Distribution. This information helps to develop Knowledge Base (KB). This KB serves answer for all queries for the given problem.

Consider Boolean Variables :-

1) Toothache :- Pain in the tooth
2) Cavity :- Damaged tooth due to infection
3) Catch :- Dentist hold a tooth with steel probe.

	Toothache		¬Toothache	
	catch	¬catch	catch	¬catch
cavity	0.108 (10%)	0.012 (1%)	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576 (57%)

P(Cavity) = 0.108+0.12+0.72+0.08 = 0.2

P(cavity ∧ Toothache) = 0.108 + 0.012

P(cavity ∨ Toothache) = 0.108 + 0.012 + 0.072 + 0.008 + 0.016 + 0.064 = 0.28

Conditional probability

$$P(\text{cavity} \mid \text{Toothache}) = \frac{P(\text{cavity} \wedge \text{Toothache})}{P(\text{Toothache})} = \frac{0.108+0.92}{0.108+0.012+0.016+0.064} = \boxed{0.6}$$

$$P(\neg\text{cavity} \mid \text{Toothache}) = \frac{P(\neg\text{cavity} \wedge \text{Toothache})}{P(\text{Toothache})} = \frac{0.016+0.064}{0.108+0.012+0.016+0.064} = \boxed{0.4}$$

→ **Independence** :- Full Joint Distribution with the variables Toothache, Cavity and catch.

(weather)

Sunny	Toothache		¬Toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Now added another variable weather which totally independent of above variables can be given as.

Above table is for sunny. we can have 3 more table for Rain, cloudy, Snow.

From the table with 8 entries we can generate a large table with 32 entries. for 4 weather conditions.

Since the influence of toothache on weather is not true we say they are independent. we conclude

$$P(\text{cloudy} \mid \text{Toothache}, \text{catch}, \text{cavity}) = P(\text{cloudy})$$

Probability Distribution :-

$$P(\text{cloudy}, \text{Toothache}, \text{catch}, \text{cavity}) = P(\text{cloudy})\, P(\text{Toothache}, \text{catch}, \text{cavity})$$

Due to independency. Probability can ~~not~~ be the product of Individual domains.

$P(\text{Cloudy}, \text{Toothache}, \text{catch}, \text{cavity}) = P(\text{Cloudy})\, P(\text{Toothache}, \text{catch}, \text{cavity})$

This property is absolute independence.

That is a is weather and b is dental then

1) $P(a|b) = P(a)$

→ Probability of a given b is same as probability of a because they are independent.

2) $P(b|a) = P(b)$

→ Probability of b given a is same as probability of b because they are independent.

3) $P(a \wedge b) = P(a)\, P(b)$

→ Probability of a AND b is same as their indiuical Probability because they are independent.