

MODULE - 01

Fundamentals of Logic.

Introduction:- Logic - A Method of Reasoning
Logic is expressed in a symbolic language is
called Mathematical logic.

(*) PROPOSITIONS:-

A proposition is a statement (declaration)
which in a given context, can be said to be either
true (or) false, but not both.

- Ex:- 1. Bangalore is in Karnataka
2. 7 is divisible by 3 .
3. $xy = yx$, for this we cannot decisively
say whether it is true (or) not, unless we
know that what is x and y (whether it is integer
(or) Real no) so it is not a proposition.

- propositions are usually represented by small
letters such as p, q, r, s , etc.
- The truth (or) the falsity of a proposition is
called its truth table value.
- If a proposition is true it is denoted by the
truth value 1

→ If a proposition is false it is denoted by the truth value '0'.

(*) Logical connectives :-

The words like 'not', 'and', 'if then', 'if and only if', such words are called as logical connectives.

(*) Compound proposition :-

The new proposition is obtained by combining the two given propositions using the logical connectives are called as compound proposition.

(*) Simple proposition :-

Proposition which do not contain any logical connectives are called simple propositions.

(*) Negation ($\sim$)

A proposition is obtained by inserting the word 'not' in an appropriate place is called the negation of the given proposition.

The negation of a proposition 'p' is denoted by $\sim p$.

eg:- 1) p : 3 is a prime number — 1

$\sim p$: 3 is not a prime number — 0

Truth table:

p	$\sim p$
1	0
0	1

(2) p : 8 is divisible by 3 - 0

$\sim p$: 8 is not divisible by 3 - 1

Truth table

p	$\sim p$
0	1
1	0

(*) Conjunction ($\wedge$)

A compound proposition is obtained by inserting the word 'and' between two given propositions is called conjunction of the given proposition.

The conjunction of p and q is denoted by $p \wedge q$.

The conjunction of p and q is true only when both p and q are true, in all other cases it is false.

Truth table:

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

(*) Disjunction ($\vee$)

A compound proposition is obtained by inserting the word 'or' between two given proposition is called disjunction of the given proposition.

The disjunction of p and q is denoted by $p \vee q$.

Truth table:

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

(*) The disjunction of p and q

(*) The disjunction $p \vee q$ is false only when only both p & q are false otherwise it is true

(*) Exclusive disjunction ($\underline{\vee}$)

The exclusive disjunction of two propositions p and q is denoted by $p \underline{\vee} q$ (Read it as either p or q)

The exclusive disjunction is true only when either p is true or q is true but not both

Truth table:

p	q	$p \underline{\vee} q$
0	0	0
0	1	1
1	0	1
1	1	0

(*) conditional ($\rightarrow$)

A compound proposition is obtained by inserting the word 'if then' in an appropriate place is called the conditional.

The conditional of p and q is denoted by $p \rightarrow q$.
(read as if p then q).

The conditional $p \rightarrow q$ is false only when p is true and q is false.

Truth table:

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

(*) Bi-conditional ($\leftrightarrow$)

A compound proposition is obtained by inserting the word 'if and only if' in an appropriate place is called as biconditional of the given proposition.

It is denoted by $p \leftrightarrow q$.

The biconditional $p \leftrightarrow q$ is true only when both p and q have the same truth values otherwise it is false.

The biconditional of p and q is denoted by

$$p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$$

Truth table :

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

combined truth table :-

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \vee q$	$p \vee \sim q$	$p \rightarrow q$	$p \leftrightarrow q$
0	0	1	1	0	0	0	1	1
0	1	1	0	0	1	1	1	0
1	0	0	1	0	1	1	0	0
1	1	0	0	1	1	0	1	1

$q \rightarrow p$
1
0
1
1

PROBLEMS:

1. Let p : A circle is a conic

q : $\sqrt{5}$ is a real number.

r : Exponential series is convergent.

Express the following compound propositions in words

(i) $p \wedge (\sim q)$ (ii) $(\sim p) \vee q$ (iii) $p \vee (\sim q)$ (iv) $q \rightarrow (\sim p)$

(v) $p \rightarrow (q \vee r)$ (vi) $\sim p \leftrightarrow q$

Soln:- (i) $p \wedge (\sim q)$:

A circle is a conic and $\sqrt{5}$ is not a real no.

(ii) $(\sim p) \vee q$:

A circle is not a conic or $\sqrt{5}$ is a real number.

(iii) $p \vee (\sim q)$:

Either a circle is a conic or $\sqrt{5}$ is a real number.
(but not both)

(iv) $q \rightarrow (\sim p)$:

If $\sqrt{5}$ is a real number, then a circle is not a conic.

(v) $p \rightarrow (q \vee r)$:

If a circle is a conic then either $\sqrt{5}$ is a real no
or the exponential series is convergent (but not both).

(vi) $\sim p \leftrightarrow q$

If a circle is not a conic then $\sqrt{5}$ is a real number
and if $\sqrt{5}$ is a real number then a circle is not a
conic.

(2) Construct the truth tables for the following compound propositions:

(i) $p \wedge (\sim q)$ (ii) $(\sim p) \vee q$ (iii) $p \rightarrow (\sim q)$ (iv) $(\sim p) \vee (\sim q)$

:-

p	q	$\sim p$	$\sim q$	$p \wedge (\sim q)$	$(\sim p) \vee q$	$p \rightarrow (\sim q)$	$(\sim p) \vee (\sim q)$
0	0	1	1	0	1	1	0
0	1	1	0	0	1	1	1
1	0	0	1	1	0	1	1
1	1	0	0	0	1	0	0

(3) Let p , q and r be the propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions.

(i) $(p \vee q) \vee r$

(ii) $(p \wedge q) \rightarrow r$

(iii) $p \rightarrow [q \rightarrow (\sim r)]$

Soln:- (i) $(p \vee q) \vee r = (0 \vee 0) \vee 1$
 $= 0 \vee 1$
 $= 1$

(ii) $(p \wedge q) \rightarrow r = (0 \wedge 0) \rightarrow 1$
 $= 0 \rightarrow 1$
 $= 1$

(iii) $p \rightarrow [q \rightarrow (\sim r)] = 0 \rightarrow [0 \rightarrow (0)]$
 $= 0 \rightarrow 1$
 $= 1$

(4) construct the truth table for the following compound propositions.

(i) $(p \wedge q) \rightarrow (\sim r)$

(ii) $q \wedge (\sim r \rightarrow p)$

soln:-

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \rightarrow (\sim r)$	$\sim r \rightarrow p$	$q \wedge (\sim r \rightarrow p)$
0	0	0	0	1	1	0	0
0	0	1	0	0	1	1	0
0	1	0	0	1	1	0	0
0	1	1	0	0	1	1	1
1	0	0	0	1	1	1	0
1	0	1	0	0	1	1	0
1	1	0	1	1	1	1	1
1	1	1	1	0	0	1	1

(*) Tautology :

A compound proposition which is true for all possible truth values of its components is called a tautology.

(*) Contradiction :

A compound proposition which is false for all possible truth values of its components is called a contradiction.

(*) Contingency :

A compound proposition that can be true or false (depending upon the truth values of its components) is called a contingency.

Note:- Contingency is a compound proposition which is neither a tautology nor a contradiction.

PROBLEMS:

(1) prove that for any proposition p , $p \vee \sim p$ is tautology and $p \wedge \sim p$ is a contradiction.

:-

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
0	1	1	0
1	0	1	0

Hence $p \vee \sim p$ is a tautology and $p \wedge \sim p$ is a contradiction.

(2) Show that, for any propositions p and q
 the compd. proposition $p \rightarrow (p \vee q)$ is a tautology
 and the compd. proposition $p \wedge (\sim p \wedge q)$ is a contradiction

:-

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
0	0	0	1	1	0	0
0	1	1	1	1	1	0
1	0	1	1	0	0	0
1	1	1	1	0	0	0

Hence, $p \rightarrow (p \vee q)$ is true for ^{all} possible truth value

hence it is tautology

$p \wedge (\sim p \wedge q)$ is false for all possible truth values

hence it is contradiction.

(3) prove that for any propositions p, q, r then
 the compound proposition $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$
 is a tautology.

:-

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \rightarrow (p \rightarrow r)$	$\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	1	1	0	0	1	1
1	0	1	1	1	0	1	1	1
1	1	0	0	0	1	0	0	1
1	1	1	1	1	1	1	1	1

^{imp} (4) prove that for any proposition p, q, r the compound proposition.

$[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$ is tautology.

:-

p	q	r	$p \vee q$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$	$(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}$	$[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$
0	0	0	0	1	1	1	0	1
0	0	1	0	1	1	1	0	1
0	1	0	1	1	0	0	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

Hence, $[(p \vee q) \wedge \{(p \rightarrow r) \wedge (q \rightarrow r)\}] \rightarrow r$ is a tautology.

- (5) prove that
- (i) $(p \vee q) \vee (p \leftrightarrow q)$ is a tautology
 - (ii) $(p \vee q) \wedge (p \leftrightarrow q)$ is a contradiction
 - (iii) $(p \vee q) \wedge (p \rightarrow q)$ is a contingency.

:-

p	q	$p \vee q$	$p \leftrightarrow q$	$p \rightarrow q$	(i)	(ii)	(iii)
0	0	0	1	1	1	0	0
0	1	1	0	1	1	0	1
1	0	1	0	0	1	0	0
1	1	1	1	1	1	1	1

Logical Equivalence ($\Leftrightarrow$)

Two propositions u and v are said to be logically equivalent whenever u and v have the same truth value.

We write, $u \Leftrightarrow v$. Here the symbol $\Leftrightarrow$ stands for "logically equivalent to".

Note:- 1. When the propositions u and v are not logically equivalent, we write $u \not\Leftrightarrow v$.

2. Whenever u and v are logically equivalent then $u \Leftrightarrow v$ is always tautology.

problems:-

(1) prove that for any three propositions p, q, r

$$[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)].$$

Soln:-

p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

It is logically equivalent.

(Q) Examine whether compound proposition

$$[(p \vee q) \rightarrow r] \Leftrightarrow [\sim r \rightarrow \sim(p \vee q)]$$

Soln:-

p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$\sim r$	$\sim(p \vee q)$	$\sim r \rightarrow \sim(p \vee q)$
0	0	0	0	1	1	1	1
0	0	1	0	1	0	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	0	0	1
1	0	0	1	0	1	0	0
1	0	1	1	1	0	0	1
1	1	0	1	0	1	0	0
1	1	1	1	1	0	0	1

Hence, It is logically equivalent.

(*) The laws of Logic

Try

1. prove that, for any three propositions p, q, r

$$[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$$

2. show that the compound propositions $p \wedge (\sim q \vee r)$

and $p \vee (q \wedge \sim r)$ are logically equivalent.

(*) The Laws of Logic

In the following laws, T_0 denotes a tautology and F_0 denotes a contradiction.

1. Law of Double negation.

$$(\sim\sim p) \Leftrightarrow p$$

2. Idempotent law

$$(i) (p \vee p) \Leftrightarrow p \quad (ii) (p \wedge p) \Leftrightarrow p$$

3. Identity law

$$(i) (p \vee F_0) \Leftrightarrow p \quad (ii) (p \wedge T_0) \Leftrightarrow p$$

~~4.~~ Inverse law

$$(i) (p \vee \sim p) \Leftrightarrow T_0 \quad (ii) (p \wedge \sim p) \Leftrightarrow F_0$$

~~5.~~ Domination laws.

$$(i) (p \vee T_0) \Leftrightarrow T_0 \quad (ii) (p \wedge F_0) \Leftrightarrow F_0$$

6. Commutative Law

$$(i) (p \vee q) \Leftrightarrow (q \vee p) \quad (ii) (p \wedge q) \Leftrightarrow (q \wedge p)$$

~~7.~~ Absorption law

$$(i) [p \vee (p \wedge q)] \Leftrightarrow p$$
$$(ii) [p \wedge (p \vee q)] \Leftrightarrow p$$

107
 (8) De-Morgan's Law

$$(i) \sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$$

$$(ii) \sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$$

(9) Associative Law

$$(i) p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$$

$$(ii) p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$$

(10) Distributive Law

$$(i) p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$$

$$(ii) p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$$

(11) Law for the conditional

$$p \rightarrow q \Leftrightarrow \sim p \vee q$$

(*) (a) proof.

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

(*) (i) proof.

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Hence proof

PROBLEMS:

(1) prove the following logical equivalence without using truth table.

$$(i) \quad p \vee [p \wedge (p \vee q)] \Leftrightarrow p.$$

Soln: $\therefore p \vee [p \wedge (p \vee q)] \Leftrightarrow p \vee p$, by Absorption law
 $\Leftrightarrow p$, by an idempotent law.

$$(ii) \quad [p \vee q \vee (\sim p \wedge \sim q \wedge r)] \Leftrightarrow (p \vee q \vee r)$$

Soln: $[p \vee q \vee (\sim p \wedge \sim q \wedge r)]$

$$\Leftrightarrow [p \vee q \vee (\sim(p \vee q) \wedge r)] \quad \text{by De-morgan's law}$$

$$\Leftrightarrow [(p \vee q) \vee (\sim(p \vee q) \wedge r)]$$

$$\Leftrightarrow [(p \vee q) \vee \sim(p \vee q)] \wedge [(p \vee q) \vee r] \quad (\because \text{by distributive property})$$

$$\Leftrightarrow 1 \wedge [p \vee q \vee r] \quad (\text{by inverse law \& Associativity})$$

$$\Leftrightarrow (p \vee q \vee r) \quad (\text{by identity})$$

$$(iii) \quad [(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$$

Soln: consider,

$$[(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)]$$

$$\Leftrightarrow [\sim(p \wedge q) \rightarrow (p \wedge q \wedge r)]$$

$$(p \wedge q) \vee (p \wedge q \wedge r)$$

(WKT

$$p \rightarrow q \Leftrightarrow \sim p \vee q$$

$$\Leftrightarrow (p \wedge q) \vee [(p \wedge q) \wedge 0]$$

WKT Absorption Law

$$\Leftrightarrow p \wedge q //$$

$$p \vee (p \wedge q) \Leftrightarrow p.$$

(2) prove the following logical equivalences:

$$(i) [(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$$

$$\text{soln:- } [(p \vee q) \wedge (p \vee \sim q)] \vee q$$

$$\Leftrightarrow [p \vee (q \wedge \sim q)] \vee q$$

$$\left[\begin{array}{l} \text{WKT distributive law} \\ p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r) \end{array} \right]$$

$$\Leftrightarrow [p \vee F_0] \vee q$$

$$\Leftrightarrow p \vee q //$$

$$(ii) (p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim(q \vee p)$$

$$\text{soln:- } (p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)]$$

$$\Leftrightarrow (p \rightarrow q) \wedge [\sim q \wedge (\sim q \vee r)] \quad \text{by commutative}$$

$$\Leftrightarrow (p \rightarrow q) \wedge [\sim q] \quad \because \text{by Absorption law.}$$

$$\Leftrightarrow (\sim p \vee q) \wedge \sim q$$

$$\Leftrightarrow (\sim p \wedge \sim q) \vee (q \wedge \sim q)$$

$$\Leftrightarrow \sim(p \vee q) \vee F_0$$

$$\Leftrightarrow \sim(p \vee q)$$

$$\Leftrightarrow \sim(q \vee p) //$$

$$(iii) p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$$

$$\begin{aligned} \therefore p \rightarrow (q \rightarrow r) &\Leftrightarrow p \rightarrow (\sim q \vee r) \\ &\Leftrightarrow \sim p \vee (\sim q \vee r) \\ &\Leftrightarrow (\sim p \vee \sim q) \vee r \\ &\Leftrightarrow \sim(p \wedge q) \vee r \\ &\Leftrightarrow (p \wedge q) \rightarrow r // \end{aligned}$$

$$(iv) [\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$$

$$\text{soln:- } [\sim p \wedge (\sim q \wedge r)] \vee (q \wedge r) \vee (p \wedge r)$$

$$\Leftrightarrow [(\sim p \wedge \sim q) \wedge r] \vee (q \wedge r) \vee (p \wedge r)$$

$$\Leftrightarrow [\sim(p \vee q) \wedge r] \vee (q \wedge r) \vee (p \wedge r)$$

$$\Leftrightarrow [\sim(p \vee q) \wedge r] \vee [(q \vee p) \wedge r]$$

$$\Leftrightarrow [\sim(p \vee q) \vee (q \vee p)] \wedge r$$

$$\Leftrightarrow [\sim(p \vee q) \vee (p \vee q)] \wedge r$$

$$\Leftrightarrow [T] \wedge r$$

$$\Leftrightarrow r //$$

$$(v) \sim[\sim[(p \vee q) \wedge r] \rightarrow \sim q] \Leftrightarrow \sim[\sim[(p \vee q) \wedge r] \vee \sim q] \Leftrightarrow (p \vee q) \wedge r$$

$$\text{soln:- } \sim[\sim[(p \vee q) \wedge r] \rightarrow \sim q]$$

$$\Leftrightarrow \sim[\sim[(p \vee q) \wedge r] \vee \sim q]$$

$$\Leftrightarrow [(p \vee q) \wedge r] \wedge q$$

by demorgan's law
(by double negation)

$$\Leftrightarrow [p \vee (q \wedge r)] \wedge q$$

$$\Leftrightarrow [(p \vee q) \wedge r] \wedge q$$

$$\Leftrightarrow [(p \vee q) \wedge (r \wedge q)] \quad (\text{Associative law})$$

$$\Leftrightarrow (p \vee q) \wedge (q \wedge r) \quad \text{commutative law.}$$

$$\Leftrightarrow [(p \vee q) \wedge q] \wedge r \quad \text{Absorption law.}$$

$$\Leftrightarrow q \wedge r //$$

(3) prove that

$$[(p \vee q) \wedge \sim\{\sim p \wedge (\sim q \vee \sim r)\}] \vee (\sim p \wedge \sim q) \text{ is a } \vee (\sim p \wedge \sim r)$$

tautology

Soln:- Let W denote the given proposition,

$$W = [(p \vee q) \wedge \sim\{\sim p \wedge (\sim q \vee \sim r)\}] \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$$

$$W = u \vee v$$

Now $u \Leftrightarrow [(p \vee q) \wedge \sim\{\sim p \wedge (\sim q \vee \sim r)\}]$

$$\Leftrightarrow [(p \vee q) \wedge \sim\{\sim p \wedge \sim(q \wedge r)\}] \quad \text{by demorgan's}$$

$$\Leftrightarrow [(p \vee q) \wedge \sim\{\sim(p \vee (q \wedge r))\}] \quad \text{Again demorgan's}$$

$$\Leftrightarrow [(p \vee q) \wedge (p \vee (q \wedge r))] \quad \text{by double negation}$$

$$\Leftrightarrow p \vee \{q \wedge (q \wedge r)\} \quad \text{by}$$

$$\Leftrightarrow p \vee \{(q \wedge q) \wedge r\}$$

$$\Leftrightarrow p \vee (q \wedge r)$$

$$v \Leftrightarrow \sim(p \vee q) \vee \sim(p \vee r)$$

$$\Leftrightarrow \sim\{(p \vee q) \wedge (p \vee r)\} \quad \text{by demorgan's law.}$$

$$\Leftrightarrow \sim\{p \vee (q \wedge r)\}$$

$$\text{Now } (1) \Rightarrow w \Leftrightarrow u \vee v$$

$$\Leftrightarrow \{p \vee (q \wedge r)\} \vee \sim\{p \vee (q \wedge r)\}$$

$$\Leftrightarrow T_0 //$$

Thus the given compound proposition is a tautology.

(*) Simplify the following compound proposition using the laws of logic.

$$(a) (p \vee q) \wedge [\sim\{(\sim p \wedge q)\}]$$

$$\Leftrightarrow (p \vee q) \wedge [\sim\sim p \vee \sim q] \quad \text{by demorgan's law}$$

$$\Leftrightarrow (p \vee q) \wedge [p \vee \sim q]$$

$$\Leftrightarrow p \vee (q \wedge \sim q) \quad (\text{by distributive law})$$

$$\Leftrightarrow p \vee (F_0) \quad (\text{by inverse law})$$

$$\Leftrightarrow p // \quad (\text{by identity law})$$

$$(b) \sim[\sim\{(p \vee q) \wedge r\} \vee \sim q]$$

$$\Leftrightarrow \sim[\sim\{(p \vee q) \wedge r\} \vee \sim q]$$

$$\sim\sim[(p \vee q) \wedge r] \wedge \sim\sim q \quad \text{demorgan's law}$$

$$\Leftrightarrow [(p \vee q) \wedge r] \wedge q$$

$$\Leftrightarrow (p \vee q) \wedge (r \wedge q) \quad \text{distributive law.}$$

$$\Leftrightarrow (p \vee q) \wedge (q \wedge r) \quad \text{by commutative law}$$

$$\Leftrightarrow [(p \vee q) \wedge q] \wedge r \quad \text{Associative law}$$

$$\Leftrightarrow q \wedge r \quad \text{by Absorption law.}$$

(c) Let x be a specified no. write down the negation of the following conditional
 "If x is an integer, then x is a rational no."

Soln:- It is of the form $p \rightarrow q$

where, p : x is an integer

q : x is a rational number.

$$\sim(p \rightarrow q) \Leftrightarrow \sim(\sim p \vee q)$$

$$\Leftrightarrow \sim\sim p \wedge \sim q$$

demorgan's law

$$\Leftrightarrow p \wedge \sim q$$

i.e. " x is an integer and x is not a rational no."

(d) Let x be a specified no write down the negation of the following proposition:

Soln:- "If x is not a real no, then it is not a rational no and not an irrational no".

Soln:- p : x is a real number

q : x is a rational number

r : x is a irrational no

The given statement is of the form,

$$\sim p \rightarrow (\sim q \wedge \sim r)$$

Now, $\sim[\sim p \rightarrow (\sim q \wedge \sim r)]$

$$\Leftrightarrow \sim[\sim p \rightarrow \{\sim(q \vee r)\}]$$

$$\Leftrightarrow \sim[\sim\sim p \vee \sim(q \vee r)]$$

$$\Leftrightarrow \sim(p \vee \sim(q \vee r)) \quad \text{demorgan's law}$$

$$\Leftrightarrow \sim p \wedge \sim\sim(q \vee r)$$

$$\Leftrightarrow \sim p \wedge (q \vee r)$$

$\therefore x$ is not a real no and it is a rational no or irrational no.

(*) Duality

Suppose 'u' is a compound proposition and its duality is obtained by replacing

(i) each $\wedge$ and $\vee$ by $\vee$ and $\wedge$ respectively

(ii) each T_0 & F_0 by F_0 & T_0 respectively

And it is denoted by u^d .

ex:- $u: p \wedge (q \vee \sim r) \vee (s \wedge T_0)$

$u^d: p \vee (q \wedge \sim r) \wedge (s \vee F_0)$

(*) Converse, Inverse and Contrapositive:

consider a conditional $p \rightarrow q$ then

(1) $q \rightarrow p$ is called the converse of $p \rightarrow q$

(2) $\sim p \rightarrow \sim q$ is called the inverse of $p \rightarrow q$

(3) $\sim q \rightarrow \sim p$ is called the contrapositive of $p \rightarrow q$.

(*) RULE OF INFERENCE:

consider a set of propositions $P_1, P_2, P_3, \dots, P_n$ and a proposition Q . Then a compound proposition of the form $(P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n) \rightarrow Q$ is called a argument. Here $P_1, P_2, \dots, P_n$ are called the premises of the argument and Q is called a conclusion of the argument.

It is represented in the form of tabular form, i.e.

$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \vdots \\ P_n \\ \hline \therefore Q \end{array}$$

$\Rightarrow$ The preceding argument is said to be valid if whenever each of the premises $P_1, P_2, \dots, P_n$ is true, then the conclusion Q is likewise true.

In other words, the argument

$(P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n) \rightarrow Q$ is valid when

$$(P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n) \Rightarrow Q$$

For finding the validity of Argument use the Rules of Logic and these rules are called the Rules of Inference.

(1) Rule of conjunctive simplification.

For any two propositions p and q , if $p \wedge q$ is true then p is true

$$\text{i.e. } p \wedge q \Rightarrow p$$

2) Rule of Disjunctive Amplification

for any two propositions p and q , if p is true then $p \vee q$ is true i.e.

$$p \Rightarrow p \vee q$$

3) Rule of Syllogism:

for any three propositions, p, q, r
if $p \rightarrow q$ is true and $q \rightarrow r$ is true then $p \rightarrow r$ is true

$$\text{i.e. } \{ (p \rightarrow q) \wedge (q \rightarrow r) \} \Rightarrow p \rightarrow r$$

In tabular form

$$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

4) Modus ponens

this rule states that if p is true and $p \rightarrow q$ is true, then q is true

$$\text{i.e. } \{ p \wedge (p \rightarrow q) \} \Rightarrow q$$

In tabular form

$$\begin{array}{c} p \\ p \rightarrow q \\ \hline \therefore q \end{array}$$

5) Modus Tollens :

This rule states that if $p \rightarrow q$ is true and q is false, then p is false

$$\text{i.e. } \{(p \rightarrow q) \wedge \sim q\} \Rightarrow \sim p$$

In tabular form,

$$\begin{array}{c} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

6) Rule of Disjunctive Syllogism :

This rule states that if $p \vee q$ is true and p is false, then q is true

$$\text{i.e. } [(p \vee q) \wedge \sim p] \Rightarrow q$$

In tabular form,

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

PROBLEMS: Test whether the following are valid argument.

(1) If Sachin hits a century, then he gets a free car.

Sachin hits a century

$\therefore$ Sachin gets a free car.

Soln:- Let p : Sachin hits a century

q : Sachin gets a free car.

The given argument is of the form

$$\begin{array}{c} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

$$[(p \rightarrow q) \wedge p] \Rightarrow q$$

$$\Leftrightarrow [p \wedge (p \rightarrow q)] \Rightarrow q \quad (\text{By Modus ponens})$$

This is a valid argument.

(2) If Sachin hits a century, he gets a free car

Sachin does not get a free car

$\therefore$ Sachin has not hit a century

Soln:- Let p : Sachin hits a century

q : Sachin gets a free car

The given argument is of the form

$$\begin{array}{c} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

$$[(p \rightarrow q) \wedge \sim q] \Rightarrow \sim p \quad (\text{by Modus Tollens})$$

This is a valid argument.

(3) If Sachin hits a century, he gets a free car.

Sachin gets a free car

$\therefore$ Sachin has hit a century.

Soln:- Let p : Sachin hits a century

q : Sachin gets a free car.

The given argument is

$$\begin{array}{c} p \rightarrow q \\ q \\ \hline \therefore p \end{array}$$

$$[(p \rightarrow q) \wedge q] \Rightarrow p$$

$$\Leftrightarrow [(\sim p \vee q) \wedge q] \quad \text{~~not~~}$$

$$\Leftrightarrow q \quad (\text{absorption law})$$

$\therefore$ The given argument is not a valid.

(4) If I drive to work, then I will arrive tired
 I am not tired (when I arrive at work)

 $\therefore$ I do not drive to work.

Soln:- let p : I drive to work
 q : I arrive tired.

The given argument is of the form

$$\begin{array}{c} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

$$[(p \rightarrow q) \wedge \neg q] \Rightarrow \neg p \quad [\text{by Modus Tollens}]$$

It is a valid argument.

(5) I will become famous (or) I will not become a Musician

I will become a musician

$\therefore$ I will become famous.

Soln:- let p : I will become famous
 q : I will become a musician

The given argument is

$$\begin{array}{c} p \vee \neg q \\ \cdot \quad q \\ \hline \therefore p \end{array}$$

$$[(p \vee \sim q) \wedge q] \Rightarrow p$$

$$\Leftrightarrow (q \rightarrow p) \wedge q$$

$$\Leftrightarrow q \wedge (q \rightarrow p) \Rightarrow p$$

WKT by

Modus ponens rule

$$p \wedge (p \rightarrow q) \Rightarrow q$$

Hence it is a valid argument.

(6) If I study, then I do not fail in the examination.
If I do not fail in the examination, my father gifts a two-wheeler to me.

$\therefore$ if I study then my father gifts a two-wheeler to me.

Soln:- Let p : I study

q : I do not fail in the examination

r : My father gifts a two-wheeler to me.

The given argument reads

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow p \rightarrow r \quad (\text{by Rule of Syllogism})$$

It is a valid argument.

(7) If Ravi goes out with friends, he will not study
 If Ravi does not study, his father becomes angry
 His father is not angry

$\therefore$ Ravi has not gone out with friends.

Soln:- Let p : Ravi goes out with friends

q : Ravi does not study

r : His father becomes angry

The given argument is

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \sim r \\ \hline \therefore \sim p \end{array}$$

$$[(p \rightarrow q) \wedge (q \rightarrow r) \wedge \sim r] \Rightarrow \sim p$$

$$\text{Now, } [(p \rightarrow q) \wedge (q \rightarrow r) \wedge \sim r] \quad (\text{Rule of Syllogism})$$

$$\Rightarrow [(p \rightarrow r) \wedge \sim r] \quad (\text{by Rule of Modus tollens})$$

$$\Rightarrow \sim p$$

(8) If I study, I will not fail in the examination
 If I do not watch TV in the evenings, I will study
 I failed in the examination

$\therefore$ I must have watched TV in the evenings.

Soln:- let p : I study

q : I fail in the examination

r : I watch TV in the evenings.

The given argument is

$$\begin{array}{c} p \rightarrow \sim q \\ \sim r \rightarrow p \\ q \\ \hline \therefore r \end{array}$$

$$[(p \rightarrow \sim q) \wedge (\sim r \rightarrow p) \wedge q] \Rightarrow r$$

$$\Rightarrow [(\sim r \rightarrow p) \wedge (p \rightarrow \sim q) \wedge q]$$

Rule of syllogism

$$\Rightarrow [(\sim r \rightarrow \sim q) \wedge q]$$

Modus tollens
 $[(p \rightarrow q) \wedge \sim q \Rightarrow \sim p]$

$$\Rightarrow \sim(\sim r)$$

$$\Rightarrow r$$

It is a valid argument.

(9) I will get grade A in this course or I will not graduate

If do not graduate, I will join the army

$\therefore$ I will not join the army.

Soln:- let p : I get grade A in this course

q : I do not graduate

τ : I join the army

Then the given argument is

$$\begin{array}{c} p \vee q \\ q \rightarrow \tau \\ p \\ \hline \therefore \sim \tau \end{array}$$

$$[(p \vee q) \wedge (q \rightarrow \tau) \wedge p]$$

$$\Leftrightarrow [(\sim \sim p \vee q) \wedge (q \rightarrow \tau) \wedge p] \quad \text{Law for conditional}$$

$$\Leftrightarrow [(\sim p \rightarrow q) \wedge (q \rightarrow \tau) \wedge p] \quad \text{Rule of syllogism.}$$

$$\Leftrightarrow [(\sim p \rightarrow \tau) \wedge p] \quad \text{there is no rule}$$

$$\Leftrightarrow [(\sim \sim p \vee \tau) \wedge p]$$

$$\Leftrightarrow [(p \vee \tau) \wedge p] \quad \text{by Absorption law}$$

$$\Rightarrow \tau //$$

It is not a valid.

(10) If I have talent and work hard, then I will become successful in life

If I become successful in life, then I will be happy

$\therefore$ If I will not be happy, then I did not work hard
($\because$) I do not have talent.

Soln:- let p : I have talent

q : I will work hard

τ : I will become successful in life

S : I will be happy

The given argument is $(p \wedge q) \rightarrow r$

$$r \rightarrow S$$

$$\therefore \sim S \rightarrow (\sim q \vee \sim p)$$

$$[(p \wedge q) \rightarrow r] \wedge (r \rightarrow S) \Rightarrow \sim S \rightarrow (\sim q \vee \sim p)$$

Now, $(p \wedge q) \rightarrow r \wedge (r \rightarrow S)$

Rule of Syllogism

$$\Rightarrow (p \wedge q) \rightarrow S$$

$$\Rightarrow \sim S \rightarrow \sim(p \wedge q) \quad (\text{de Morgan's})$$

$$\Rightarrow \sim S \rightarrow \sim p \vee \sim q$$

$$\Rightarrow \sim S \rightarrow \sim q \vee \sim p$$

It is valid argument.

(ii) If Ravi studies, then he will pass in Discrete Maths paper

If Ravi does not play cricket, then he will study

Ravi failed in Discrete Maths paper

$\therefore$ Ravi played cricket

Soln:- p : Ravi studies

q : Ravi will pass in Discrete Maths paper

r : Ravi play cricket.

Given argument, $p \rightarrow q$

$$\sim r \rightarrow p$$

$$\sim q$$

$$\therefore r$$

$$\text{i.e. } [(p \rightarrow q) \wedge (\neg r \rightarrow p) \wedge \neg q] \Rightarrow r$$

$$\text{Now, } [(p \rightarrow q) \wedge (\neg r \rightarrow p) \wedge \neg q]$$

$$\Leftrightarrow [(\neg r \rightarrow p) \wedge (p \rightarrow q) \wedge \neg q] \quad \text{commutative law}$$

$$\Rightarrow [(\neg r \rightarrow q) \wedge \neg q] \quad (\text{syllogism})$$

$$\Rightarrow \neg(\neg r) \quad (\text{by Modus tollens})$$

$$\Rightarrow r$$

It is a valid argument.

(2) p

$$p \rightarrow \sim q$$

$$\sim q \rightarrow \sim r$$

$$\therefore \sim r$$

soln: $p \wedge (p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim r)$

$$\Rightarrow p \wedge (p \rightarrow \sim r) \quad (\text{syllogism})$$

$$\Rightarrow \sim r \quad \text{by Modus ponens}$$

(3) $p \rightarrow r$

$$q \rightarrow r$$

$$(p \vee q) \rightarrow r$$

soln:- $(p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow (p \vee q) \rightarrow r$

$$\Leftrightarrow (\sim p \vee r) \wedge (\sim q \vee r)$$

distributive law

$$\Leftrightarrow (\sim p \wedge \sim q) \vee r$$

demorgan's law

$$\Leftrightarrow \sim(p \vee q) \vee r$$

conditional law.

$$\Rightarrow (p \vee q) \rightarrow r //$$

Q4) $p \rightarrow q$

$$r \rightarrow s$$

$$p \vee r$$

$$\therefore q \vee s$$

soln:- $(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)$

$$\Leftrightarrow (p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim \sim p \vee r)$$

$$\Leftrightarrow (p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim p \rightarrow r)$$

$$\Leftrightarrow (p \rightarrow q) \wedge (\sim p \rightarrow r) \wedge (r \rightarrow s)$$

$$\Leftrightarrow (p \rightarrow q) \wedge (\sim p \rightarrow s) \quad (\text{using contrapositive } p \rightarrow q \Leftrightarrow \sim q \rightarrow \sim p)$$

$$\Leftrightarrow (\sim q \rightarrow \sim p) \wedge (\sim p \rightarrow s) \quad (\text{syllogism})$$

$$\Leftrightarrow (\sim q \rightarrow s)$$

$$\Rightarrow q \vee s //$$

It is valid

$$(5) \quad p \rightarrow q$$

$$r \rightarrow s$$

$$\sim q \vee \sim s$$

$$\therefore \sim(p \wedge r)$$

$$\text{Soln:- } (p \rightarrow q) \wedge (r \rightarrow s) \wedge (\sim q \vee \sim s) \quad \text{law for condn}$$

$$\Leftrightarrow (p \rightarrow q) \wedge (r \rightarrow s) \wedge (q \rightarrow \sim s)$$

by commutative law

$$\Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow \sim s) \wedge (r \rightarrow s) \quad (\text{syllogism})$$

$$\Rightarrow (p \rightarrow \sim s) \wedge (r \rightarrow s) \quad \text{by contrapositive}$$

$$\Rightarrow (p \rightarrow \sim s) \wedge (\sim s \rightarrow \sim r)$$

$$\Rightarrow (p \rightarrow \sim r) \quad (\text{by syllogism})$$

$$\Rightarrow \sim p \vee \sim r$$

$$\Rightarrow \sim(p \wedge r) //$$

It is a valid argument.

$$\begin{array}{l}
 (6) \quad p \rightarrow r \\
 \sim p \rightarrow q \\
 \underline{q \rightarrow s} \\
 \therefore \sim r \rightarrow s
 \end{array}$$

Soln:- $(p \rightarrow r) \wedge (\sim p \rightarrow q) \wedge (q \rightarrow s)$ Syllogism

$$\Rightarrow (p \rightarrow r) \wedge (\sim p \rightarrow s) \quad \text{by contrapositive}$$

$$\Rightarrow (\sim r \rightarrow \sim p) \wedge (\sim p \rightarrow s)$$

$$\Rightarrow \sim r \rightarrow s // \quad \text{by Syllogism}$$

It is valid.

$$\begin{array}{l}
 (7) \quad (\sim p \vee \sim q) \rightarrow (r \wedge s) \\
 \quad \quad r \rightarrow t \\
 \quad \quad \underline{\sim t} \\
 \therefore p
 \end{array}$$

Soln:- $[(\sim p \vee \sim q) \rightarrow (r \wedge s)] \wedge (r \rightarrow t) \wedge \sim t$ by MT

$$\Rightarrow [(\sim p \vee \sim q) \rightarrow (r \wedge s)] \wedge \sim r$$

$$\Rightarrow [(\sim p \vee \sim q) \rightarrow (r \wedge s)] \wedge (\sim r \vee \sim s) \quad \text{(by disjunction)}$$

$$\Leftrightarrow [(\sim p \vee \sim q) \rightarrow (r \wedge s)] \wedge \sim(r \wedge s) \quad \text{de Morgan}$$

$$\Rightarrow \sim(\sim p \vee \sim q) \quad \text{by MT}$$

$$\Rightarrow \sim(\sim(p \wedge q))$$

$$\Rightarrow p \wedge q \Rightarrow p \quad \text{(by conjunction)}$$

$$(8) \quad p \rightarrow (q \rightarrow r)$$

$$\sim q \rightarrow \sim p$$

$$p$$

$$\therefore r$$

Soln:- $(p \rightarrow (q \rightarrow r)) \wedge (\sim q \rightarrow \sim p) \wedge p$ by commutative

$$\Rightarrow (p \rightarrow (q \rightarrow r)) \wedge p \wedge (\sim q \rightarrow \sim p)$$
 by Modus ponens.

$$\Rightarrow (q \rightarrow r) \wedge (\sim q \rightarrow \sim p)$$
 by contrapositive

$$\Rightarrow (q \rightarrow r) \wedge (p \rightarrow q)$$
 by commutative law

$$\Rightarrow (p \rightarrow q) \wedge (q \rightarrow r)$$
 by Syllogism

$$\Rightarrow p \rightarrow r$$

$$\Rightarrow r \quad (\because p \text{ is true then } r \text{ is also true})$$

$$(9) \quad \sim p \leftrightarrow q$$

$$q \rightarrow r$$

$$\sim r$$

$$\therefore p$$

Soln:- $(\sim p \leftrightarrow q) \wedge (q \rightarrow r) \wedge \sim r$ (WKT: $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$)

$$\Rightarrow (\sim p \rightarrow q) \wedge (q \rightarrow \sim p) \wedge (q \rightarrow r) \wedge \sim r$$
 by MT

$$\Rightarrow (\sim p \rightarrow q) \wedge (q \rightarrow \sim p) \wedge \sim q$$
 commutative law

$$\Rightarrow [(\sim p \rightarrow q) \wedge \sim q] \wedge (q \rightarrow \sim p)$$
 by M.T

$$\Rightarrow \sim(\sim p) \wedge (q \rightarrow \sim p)$$

$$\Rightarrow p \wedge (q \rightarrow \sim p) \Rightarrow (q \rightarrow \sim p) \wedge p \Rightarrow p$$

It is valid.

$$\begin{array}{l}
 (10) \quad p \rightarrow (q \rightarrow r) \\
 p \vee \sim s \\
 \hline q \\
 \therefore s \rightarrow r
 \end{array}$$

$$\text{Soln:- } (p \rightarrow (q \rightarrow r)) \wedge (p \vee \sim s) \wedge q \quad \text{commutative law}$$

$$\Rightarrow (p \rightarrow (q \rightarrow r)) \wedge (\sim s \vee p) \wedge q \quad \text{conditional law}$$

$$\Rightarrow (p \rightarrow (q \rightarrow r)) \wedge (s \rightarrow p) \wedge q \quad \text{commutative law}$$

$$\Rightarrow (s \rightarrow p) \wedge (p \rightarrow (q \rightarrow r)) \wedge q \quad \text{syllogism}$$

$$\Rightarrow (s \rightarrow (q \rightarrow r)) \wedge q \quad \text{law for cond}^n$$

$$\Leftrightarrow [\sim s \vee (q \rightarrow r)] \wedge q \quad \text{distributive law}$$

$$\Leftrightarrow (\sim s \wedge q) \vee [(q \rightarrow r) \wedge q] \quad \text{by M.P}$$

$$\Leftrightarrow (\sim s \wedge q) \vee r \quad \text{by conjunctive simplification}$$

$$\Leftrightarrow \sim s \vee r$$

$$\Leftrightarrow s \rightarrow r //$$

It is a valid.

$$(ii) \sup p \rightarrow (q \wedge r)$$

$$r \rightarrow s$$

$$\sim(q \wedge s)$$

$$\therefore \sim p$$

$$\underline{\text{Soln:-}} [p \rightarrow (q \wedge r)] \wedge (r \rightarrow s) \wedge \sim(q \wedge s) \text{ demorgan's law}$$

$$\Leftrightarrow [p \rightarrow (q \wedge r)] \wedge (r \rightarrow s) \wedge (\sim q \vee \sim s) \text{ by conditional law}$$

$$\Leftrightarrow [p \rightarrow (q \wedge r)] \wedge (r \rightarrow s) \wedge (q \rightarrow \sim s) \text{ by contrapositive}$$

$$\Leftrightarrow [p \rightarrow (q \wedge r)] \wedge (r \rightarrow s) \wedge (s \rightarrow \sim q) \text{ syllogism}$$

$$\Leftrightarrow [p \rightarrow (q \wedge r)] \wedge (r \rightarrow \sim q) \text{ by conditional law}$$

$$\Leftrightarrow [p \rightarrow (q \wedge r)] \wedge (\sim r \vee \sim q) \text{ demorgan's law}$$

$$\Leftrightarrow [p \rightarrow (q \wedge r)] \wedge \sim(r \wedge q) \text{ commutative law}$$

$$\Leftrightarrow [p \rightarrow (q \wedge r)] \wedge \sim(q \wedge r) \text{ by MT}$$

$$\Rightarrow \sim p //$$

It is valid.

$$(i) \sup \sim(p \vee q) \rightarrow r$$

$$r \rightarrow (s \vee t)$$

$$\sim s \wedge \sim u$$

$$\sim u \rightarrow \sim t$$

$$\therefore p$$

$$\underline{\text{Soln:-}} \quad \frac{[(\sim p \vee q) \rightarrow r] \wedge [r \rightarrow (s \vee t)] \wedge [\sim s \wedge \sim u]}{\text{Syllogism}} \quad \begin{matrix} [\sim u \rightarrow \sim t] \\ \text{Associative} \end{matrix}$$

$$\Rightarrow [(\sim p \vee q) \rightarrow (s \vee t)] \wedge \sim s \wedge [\sim u \wedge (\sim u \rightarrow \sim t)] \quad \text{by M.P}$$

$$\Rightarrow [(\sim p \vee q) \rightarrow (s \vee t)] \wedge (\sim s \wedge \sim t) \quad \text{demorgan's law}$$

$$\Rightarrow \underline{[(\sim p \vee q) \rightarrow (s \vee t)] \wedge \sim (s \vee t)} \quad \text{by M.T}$$

$$\Rightarrow \sim(\sim p \vee q) \quad \text{demorgan's}$$

$$\Rightarrow \sim\sim p \wedge \sim q$$

$$\Rightarrow p \wedge \sim q \quad \text{by conjunctive simplification.}$$

$$\Rightarrow p //$$

It is valid.

$$\begin{array}{l} \text{wp} \\ \text{(13)} \quad p \vee q \\ \sim p \vee r \\ \hline \sim r \\ \hline \therefore q \end{array}$$

$$\underline{\text{Soln:-}} \quad (p \vee q) \wedge \underline{(\sim p \vee r)} \wedge (\sim r) \quad \text{conditional law}$$

$$\Rightarrow \underline{(p \vee q) \wedge (p \rightarrow r)} \wedge \sim r \quad \text{by M.T}$$

$$\Rightarrow \underline{(\sim\sim p \vee q)} \wedge \sim p$$

$$\Rightarrow (\sim p \rightarrow q) \wedge \sim p \quad \text{by M.P}$$

$$\Rightarrow q // \quad \text{valid argument.}$$

$$\begin{array}{l}
 (14) \quad p \rightarrow r \\
 r \rightarrow s \\
 t \vee \sim s \\
 \sim t \vee u \\
 \hline
 \sim u \\
 \hline
 \therefore \sim p
 \end{array}$$

Soln:- $(p \rightarrow r) \wedge (r \rightarrow s) \wedge (t \vee \sim s) \wedge (\sim t \vee u) \wedge \sim u$

syllogism
commutative and conditional
conditional

$$\Rightarrow (p \rightarrow s) \wedge (s \rightarrow t) \wedge (t \rightarrow u) \wedge \sim u$$

syllogism

$$\Rightarrow (p \rightarrow s) \wedge (s \rightarrow u) \wedge \sim u$$

syllogism

$$\Rightarrow (p \rightarrow u) \wedge \sim u$$

by MT

$$\Rightarrow \sim p //$$

It is valid

$$\begin{array}{l}
 (15) \quad p \rightarrow q \\
 q \rightarrow (r \wedge s) \\
 \sim r \vee (\sim t \vee u) \\
 p \wedge t \\
 \hline
 \therefore u
 \end{array}$$

Soln:- $(p \rightarrow q) \wedge (q \rightarrow (r \wedge s)) \wedge (\sim r \vee (\sim t \vee u)) \wedge (p \wedge t)$

syllogism

$$\Rightarrow [p \rightarrow (r \wedge s)] \wedge [\sim r \vee (\sim t \vee u)] \wedge (p \wedge t)$$

$$\Rightarrow [p \rightarrow (r \wedge s)] \wedge [\underbrace{(\sim r \vee \sim t)}_{\text{demorgan's}} \vee u] \wedge (p \wedge t) \quad \text{commutative}$$

$$\Rightarrow [p \rightarrow (r \wedge s)] \wedge (p \wedge t) \wedge [\underbrace{\sim(r \wedge t) \vee u}_{\text{conditional}}]$$

$$\Rightarrow [p \rightarrow (r \wedge s)] \wedge (p \wedge t) \wedge [(r \wedge t) \rightarrow u]$$

$$\Rightarrow \underbrace{(p \rightarrow (r \wedge s)) \wedge p \wedge t}_{\text{by M.P.}} \wedge [(r \wedge t) \rightarrow u]$$

$$\Rightarrow \underbrace{(r \wedge s) \wedge t}_{\text{commutative}} \wedge [(r \wedge t) \rightarrow u]$$

$$\Rightarrow \underbrace{(r \wedge t) \wedge s}_{\text{commutative}} \wedge [(r \wedge t) \rightarrow u]$$

$$\Rightarrow \underbrace{(r \wedge t) \wedge [(r \wedge t) \rightarrow u]}_{\text{by M.P.}} \wedge s$$

$$\Rightarrow u \wedge s \quad \text{by conjunctive simplification}$$

$$\Rightarrow u$$

Try

(1) p

$$p \rightarrow q$$

$$s \vee r$$

$$r \rightarrow \sim q$$

$$\therefore s$$

(2) $C \vee D$

$$(C \vee D) \rightarrow \sim H$$

$$\sim H \rightarrow (A \wedge \sim B)$$

$$(A \wedge \sim B) \rightarrow R \vee s$$

$$\therefore R \vee s$$

OPEN STATEMENTS ; QUANTIFIERS STATEMENTS

(*) Open Statements

The declarative sentences like $x+2=3$, $x=\sqrt{2}$, $x>0$, x divisible by 2, in these sentences ' x ' is not defined, such statements are called as open statement and ' x ' is called the free variables.

→ This open statement can be converted into propositions by giving the values to ' x ' from universal set (U)

→ Open statements are denoted by $p(x)$, $q(x)$, $r(x)$ and so on.

consider $x+3=6$ and the set of real number R .
Now, this sentence becomes a proposition and if x is replaced by any element of R .

For example :-

if x is replaced by 3 → true proposition
and if x is replaced by 5, it becomes a false proposition, here we say R is a universe (or universe of discourse).

Example (1): Suppose the universal set consists of all integers, consider the following open statements
 $p(x): x \leq 3$, $q(x): x+1$ is odd, $r(x): x > 0$
Write down the truth values of the following,

(i) $p(2)$

$\therefore$ We've $p(x): x \leq 3$

$x=2, \Rightarrow p(2): 2 \leq 3$ It is true $\rightarrow 1$

(ii) $\neg q(4)$

$\therefore$ We've $q(x): x+1$ is odd

$\Rightarrow q(4): 4+1$ is odd

$\neg q(4)$ is false $\rightarrow 0$

(iii) $p(-1) \wedge q(1)$

$\therefore p(-1): -1 \leq 3 \rightarrow$ true $\rightarrow 1$

$q(1): 1+1=2$ is not odd $\rightarrow$ false $= 0$.

$\therefore p(-1) \wedge q(1) [1 \wedge 0]$ is false.

(iv) $\neg p(3) \vee r(0)$

$\therefore p(3): 3 \leq 3$ is true $= 1$

$\Rightarrow \neg p(3)$ is false $= 0$

$r(0): 0 > 0$ is false $= 0$

$\therefore \neg p(3) \wedge r(0) = 0 \wedge 0 = 0$
 $\Rightarrow$ false

(v) $p(2) \wedge [q(0) \vee \neg r(2)]$

$\therefore p(2): 2 \leq 3$ true $= 1$

$q(0): 0+1=1$ is odd, true $= 1$

$r(2): 2 > 0$

$\Rightarrow \neg r(2): 2 < 0$ is false $= 0$

$$\therefore p(2) \wedge [q(0) \vee \neg r(2)]$$

$$1 \wedge [1 \vee 0] \Rightarrow 1 \wedge 1$$

$$= 1$$

$$\Rightarrow p(2) \wedge [q(0) \vee \neg r(2)] - \text{True} - 1 //$$

Quantifiers

consider the statements

- (i) All integers are divisible by 2
- (ii) Some integers are divisible by 3
- (iii) Every integer is real
- (iv) There exists an integer which is prime

On these propositions, the words 'all', 'every', 'some', 'there exists' are associated with the idea of a quantity. Such words are called Quantifiers.

$\Rightarrow$ The symbol $\forall$ has been used to denote the phrases 'for all', 'for every', 'for any', 'for each' and these are called universal quantifiers.

$\Rightarrow$ The symbol $\exists$ has been used to denote the phrases 'there exists', 'for some', 'for atleast', these are called as existential quantifiers.

$\Rightarrow$ A proposition involving universal or existential

Quantifier is called a quantifier statement

$\Rightarrow$ Thus a quantified statement is a proposition of the form " $\forall x \in S, p(x)$ " or " $\exists x \in S, p(x)$ "

Where $p(x)$ is an open statement and S is the universe for x in $p(x)$.

"The variable present in a quantified statement is called a bound variable."

Example:- For the universe of all integers, let

$$p(x) : x > 0$$

$$q(x) : x \text{ is even}$$

$$r(x) : x \text{ is a perfect square} \quad (4, 9, 16, \dots)$$

$$s(x) : x \text{ is divisible by 3.}$$

$$t(x) : x \text{ is divisible by 7.}$$

Write down the following quantified statements in symbolic form.

(i) At least one integer is even.

$$\text{soln:- } \exists x \in S, q(x)$$

(ii) There exists a positive integer that is even

$$\text{soln:- } \exists x \in S, (p(x) \wedge q(x))$$

(iii) ~~Some even integers is either even or odd.~~

$$\text{soln:- } \exists$$

(iii) Some even integers are divisible by 3.

Soln:- $\exists x \in S, [q(x) \wedge s(x)]$

(iv) Every integers is either even (or) odd

$\therefore \forall x \in S, [q(x) \vee \sim q(x)]$

(v) If x is even and a perfect square, then x is not divisible by 3.

$\therefore \forall x \in S, [(p(x) \wedge r(x)) \rightarrow \sim s(x)]$

(vi) If x is odd (or) is not divisible by 3, then x is divisible by 3.

$\therefore \forall x \in S, [(\sim p(x) \vee \sim r(x)) \rightarrow s(x)]$

(*) Truth value of a Quantifier Statement

Rule 1:- $\forall x \in S, p(x)$ is true, then it should be true for all values of x , suppose if it is false for atleast one value, i.e. $p(a)$ is false where $a \in S$, this element 'a' is counter example, then $\forall x \in S, p(x)$ is false.

Rule 2:- For proving $\exists x \in S, p(x)$ is true, it is enough to show that $p(a)$ is true where $a \in S$.

(*) Truth value of a Quantified Statement

Rule 1: The statement " $\forall x \in S, p(x)$ " is true only when $p(x)$ is true for each $x \in S$.

Rule 2: The statement " $\exists x \in S, p(x)$ " is false^{only when $p(x)$ is false} for every $x \in S$.

Accordingly, a proposition of the form " $\forall x \in S, p(x)$ ", is false, it is enough to exhibit one element 'a' of S such that $p(a)$ is false. This element 'a' is called counter example.

Similarly a proposition of the form " $\exists x \in S, p(x)$ " is true, it is enough to exhibit one element 'a' of S s.t. $p(a)$ is true.

(*) Two Rules of Inference:

(*) If an open statement $p(x)$ is known to be true for all x in a universe S and if $a \in S$, then $p(a)$ is true (Rule of universal specification)

(*) If an open statement $p(x)$ is proved to be true for any x chosen from a set S , then the quantified statement $\forall x \in S, p(x)$ is true (Rule of universal Generalization).

problems

(1) consider the open statement $p(x): x > 0$;

$q(x): x$ is even ; $r(x): x$ is a perfect square

$s(x): x$ is divisible by 3 ; $t(x): x$ is divisible by 7

Express each of the following symbolic statements

in words. (i) $\forall x, [r(x) \rightarrow p(x)]$ (ii) $\exists x, [s(x) \wedge q(x)]$

(iii) $\forall x, [\neg r(x)]$ (iv) $\forall x, [r(x) \vee t(x)]$

Soln:- (i) For all integer x , if x is a perfect square then $x > 0$

(ii) For some integer x , x is divisible by 3 and x is odd.

(iii) For all integer x , x is not a perfect square

(iv) For all integer x , x is a perfect square (iii)
 x is divisible by 7 $\rightarrow$ False (eg:- $x = 8$)

(2) consider the following open statements with the set of all real no as the universe.

$p(x): |x| > 3$, $q(x): x > 3$, Find the truth

values of the statement $\forall x, [p(x) \rightarrow q(x)]$

Soln:- Given $p(x): |x| > 3$

for $x = -4$, i.e $p(-4): |-4| > 3$

$4 > 3$ is true - 1

And $q(x): x > 3$, for $x = -4$, $q(-4): -4 > 3$ - false - 0

For $x = -4$, $p(x) : 1$ and $q(x) : 0$

$$\therefore p(x) \rightarrow q(x) = 1 \rightarrow 0 \\ = 0$$

$\Rightarrow$ false

[We've got this result by taking $x = -4$ counter example]

(3) Consider the following open statements with the set of all real numbers as the universe

$$p(x) : x \geq 0 ; q(x) : x^2 \geq 0 ; r(x) : x^2 - 3x - 4 = 0$$

$$s(x) : x^2 - 3 > 0$$

Determine the truth values of the following statements.

- (i) $\exists x, p(x) \wedge q(x)$ (ii) $\forall x, p(x) \rightarrow q(x)$
(iii) $\forall x, q(x) \rightarrow s(x)$ (iv) $\forall x, r(x) \vee s(x)$
(v) $\exists x, p(x) \vee r(x)$ (vi) $\forall x, r(x) \rightarrow p(x)$

Soln:-

(i) there exist a real number 'x' for which both $p(x)$ and $q(x)$ are true ; for instance $x = 1$

$$\therefore p(x) \wedge q(x) = 1 \wedge 1 = 1 \rightarrow \text{true}$$

(ii) For all x , $q(x)$ is true hence $p(x) \rightarrow q(x)$ is true.

(iii) For every real no x , $s(x)$ is false for $x = 1$ and $q(x)$ is true

thus. $q(x) \rightarrow s(x)$ is false for $x = 1$

i.e. the statement $q(x) \rightarrow s(x)$ is not always true

$\therefore \forall x, q(x) \rightarrow s(x)$ is false

$$(iv) \quad r(x): x^2 - 3x - 4 = 0$$

$$(x-4)(x+1)$$

i.e. $r(x)$ is true only for $x=4$ and $x=-1$

but for all real no x , $r(x)$ is not true i.e. it is false and $s(x)$ is false for $x=1$

Hence $r(x) \wedge s(x)$ is false (not always true)

$$(v) \quad \exists x, p(x) \wedge r(x)$$

for $x=4$, both $p(x)$ and $r(x)$ is true

hence, $p(x) \wedge r(x) = 1 \wedge 1 = 1$ is true.

$$(iv) \quad \forall x, r(x) \rightarrow p(x)$$

$r(x)$ is true for $x=-1$ but $p(x)$ is false for $x=-1$

thus, $r(x) \rightarrow p(x)$ is not always true

$\therefore \forall x, r(x) \rightarrow p(x)$ is false.

(4). Let $p(x) = x^2 - 7x + 10 = 0$; $q(x) = x^2 - 2x - 3 = 0$,
 $r(x): x < 0$. Determine the truth (or) falsity of the

following statements where the universe U contains only the integers 2 and 5. If a statement is false, provide a counter example (or) explanation.

$$(i) \quad \forall x, p(x) \rightarrow \sim r(x) \quad (ii) \quad \forall x, q(x) \rightarrow r(x)$$

$$(iii) \exists x, q(x) \rightarrow r(x) \quad (iv) \exists x, p(x) \rightarrow r(x)$$

$$\text{Soln: let } U = \{2, 5\}$$

$$p(x) : x^2 - 7x + 10 = 0$$

$$q(x) : x^2 - 2x - 3 = 0$$

$$p(x) : (x-5)(x-2) = 0$$

$$q(x) : (x-3)(x+1) = 0$$

$$p(x) : x = 5, x = 2$$

$$q(x) : x = 3, x = -1$$

$$(i) \forall x, p(x) \rightarrow \sim r(x)$$

$p(x)$ is true for all $x \in U$ and $\sim r(x)$ is true for all $x \in U$

$\therefore \forall x, p(x) \rightarrow \sim r(x)$ is true.

$$(ii) \forall x, q(x) \rightarrow r(x)$$

$q(x)$ is false for all $x \in U$ and $r(x)$ is false for all $x \in U$

$\therefore \forall x, q(x) \rightarrow r(x)$ is true.

$$(iii) \exists x, q(x) \rightarrow r(x)$$

$q(x)$ and $r(x)$ is false at $x = 2$

$\therefore \exists x, q(x) \rightarrow r(x)$ is true.

$$(iv) \exists x, p(x) \rightarrow r(x)$$

$p(x)$ is true for $x = 2$ but $r(x)$ is false for $x = 2$

$\therefore \exists x, p(x) \rightarrow r(x)$ is false.

(*) Logical Equivalence :

Two quantified statements are said to be logically equivalent whenever they have the same truth values in all possible situations.

$$(i) \forall x, [p(x) \wedge q(x)] \Leftrightarrow (\forall x, p(x)) \wedge (\forall x, q(x))$$

$$(ii) \exists x, [p(x) \vee q(x)] \Leftrightarrow (\exists x, p(x)) \vee (\exists x, q(x))$$

$$(iii) \exists x, [p(x) \rightarrow q(x)] \Leftrightarrow \exists x, [\sim p(x) \vee q(x)]$$

(*) Rule for Negation of a Quantified Statement

$$(1) \sim \{ \forall x, p(x) \} \Leftrightarrow \exists x, \{ \sim p(x) \}$$

$$(2) \sim \{ \exists x, p(x) \} \Leftrightarrow \forall x, \{ \sim p(x) \}$$

PROBLEMS

(1) Consider the open statements $p(x); x > 0$,

$q(x): x$ is even, $r(x): x$ is perfect square,

$s(x): x$ is divisible by 3, $t(x): x$ is divisible by 7

Express each of the following symbolic statements in words and indicate its truth table,

$$(i) \forall x, [r(x) \rightarrow p(x)]$$

Soln. For any integer x , if x is perfect square then $x > 0$ — false (take $x=0$)

(2) Negate and simplify each of the following

(i) $\exists x, [p(x) \vee q(x)]$

soln:- $\sim \{ \exists x, [p(x) \vee q(x)] \}$

$$\Leftrightarrow \sim \exists x, p \quad \forall x, [\sim (p(x) \vee q(x))]$$

$$\Leftrightarrow \forall x, [\sim p(x) \wedge \sim q(x)] //$$

(ii) $\forall x, [p(x) \wedge \sim q(x)]$

soln:- $\sim \{ \forall x, [p(x) \wedge \sim q(x)] \}$

$$\Leftrightarrow \exists x, [\sim (p(x) \wedge \sim q(x))]$$

$$\Leftrightarrow \exists x, [\sim p(x) \vee q(x)] //$$

(iii) $\forall x, [p(x) \rightarrow q(x)]$

soln:- $\sim \{ \forall x, [p(x) \rightarrow q(x)] \}$

$$\Leftrightarrow \exists x, [\sim (p(x) \rightarrow q(x))] \quad \text{Law for cond'n.}$$

$$\Leftrightarrow \exists x, [\sim (\sim p(x) \vee q(x))] \quad \text{demorgan's}$$

$$\Leftrightarrow \exists x, [p(x) \wedge \sim q(x)]$$

(iv) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$

soln:- $\sim \{ \exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)] \}$

$$\Leftrightarrow \forall x, \{ \sim [\{p(x) \vee q(x)\} \rightarrow r(x)] \}$$

$$\Leftrightarrow \forall x, \{ \sim [\sim (p(x) \vee q(x)) \vee r(x)] \}$$

$$\Rightarrow \neg \forall x, [(p(x) \vee q(x)) \wedge \neg r(x)] //$$

✓ (3) write down the following proposition in symbolic form and find its negation

"All integers are rational numbers and some rational numbers are not integers".

:- $p(x)$: x is a rational number

$q(x)$: x is an integer

Z = Set of all integers, Q = set of all rational no

Symbolic form,

$$[\forall x \in Z, p(x)] \wedge [\exists x \in Q, \neg q(x)]$$

$$\Rightarrow \neg [\forall x \in Z, p(x)] \vee [\exists x \in Q, \neg q(x)]$$

$$\Rightarrow \neg [\forall x \in Z, p(x)] \vee \neg [\exists x \in Q, \neg q(x)]$$

$$\Rightarrow [\exists x \in Z, \neg p(x)] \vee [\forall x \in Q, q(x)] //$$

Some integers are not rational no or all rational numbers are integers.

(4) Let the set Z of all integers be the universe, consider the statements

$$p(x): 3x+1=5 \quad \text{and} \quad q(x): x^2=9$$

obtain the negation of the quantified statement

$\exists x \in Z, [p(x) \wedge q(x)]$ and express it in words.

Soln:- The negation of the given statement is

$$\sim \{ \exists x \in Z, [p(x) \wedge q(x)] \}$$

$$\Leftrightarrow \forall x \in Z, \sim [p(x) \wedge q(x)]$$

$$\Leftrightarrow \forall x \in Z, \sim p(x) \vee \sim q(x).$$

In words, For all integers x , $x+1 \neq 5$ (or) $x^2 \neq 9$.

(5) Write down the following proposition in symbolic form and find its negation.

"If all triangles are right angled, then no triangle is equiangular".

Soln:- Let S be the set of all triangle

$p(x)$: x is right angled

$q(x)$: x is equiangular.

In symbolic form,

$$[\forall x \in S, p(x)] \rightarrow [\forall x \in S, \sim q(x)]$$

Now, its negation,

$$\sim \{ [\forall x \in S, p(x)] \rightarrow [\forall x \in S, \sim q(x)] \}$$

$$\Leftrightarrow \sim \{ \sim [\forall x \in S, p(x)] \vee [\forall x \in S, \sim q(x)] \}$$

$$\Leftrightarrow [\forall x \in S, p(x)] \vee \sim [\forall x \in S, \sim q(x)]$$

$$\Leftrightarrow [\forall x \in S, p(x)] \vee [\exists x \in S, q(x)]$$

In words, For All triangles are right angled and Some triangles are equiangular.

(6) Write down the negation of each of the following statements.

- (i) For all integers 'n', if n is not divisible by 2 then n is odd
- (ii) If k, m, n are any integers where (k-m) and (m-n) are odd, then (k-n) is even.
- (iii) For all real numbers x, if $|x-3| < 7$, then $-4 < x < 10$.
- (iv) If x is a real number where $x^2 > 16$, then $x < -4$ or $x > 4$.

Soln:- 'Z' be the set of all integers and
'R' be the set of all real no

- (i) $p(x)$: n is divisible by 2
 $q(x)$: n is odd

Symbolic form, $\forall x \in Z, \sim p(x) \rightarrow q(x)$

Its negation, $\sim [\forall x \in Z, \sim p(x) \rightarrow q(x)]$

$$\Leftrightarrow \exists x \in Z, \sim (\sim p(x) \rightarrow q(x))$$

$$\Leftrightarrow \exists x \in Z, \sim [\sim \sim p(x) \vee q(x)]$$

$$\Leftrightarrow \exists x \in Z, \sim p(x) \wedge \sim q(x)$$

"For some integers n, n is not divisible by 2 and n is not odd".

(ii) $p(x): (k-m)$ is odd ; $r(x): (k-n)$ is even

$q_1(x): (m-n)$ is odd

Symbolic form, $\forall k, m, n \in \mathbb{Z}, (p(x) \wedge q_1(x)) \rightarrow r(x)$

$\therefore$ the negation of the statement is,

$$\sim \{ \forall k, m, n \in \mathbb{Z}, (p(x) \wedge q_1(x)) \rightarrow r(x) \}$$

$$\Leftrightarrow \exists k, m, n \in \mathbb{Z}, \sim [(p(x) \wedge q_1(x)) \rightarrow r(x)]$$

$$\Leftrightarrow \exists k, m, n \in \mathbb{Z}, \sim [\sim (p(x) \wedge q_1(x)) \vee r(x)]$$

$$\Leftrightarrow \exists k, m, n \in \mathbb{Z}, (p(x) \wedge q_1(x)) \wedge \sim r(x)$$

In words,

"For some integers k, m, n , $(k-m)$ and $(m-n)$ are odd and $(k-n)$ is not even".

(iii) Here, $x \in \mathbb{R}$.

$p(x): |x-3| < 7$ & $q_1(x): -4 < x < 10$ (i.e. $x \in (-4, 10)$)

Symbolic form, $\forall x \in \mathbb{R}, p(x) \rightarrow q_1(x)$

The negation is

$$\sim \{ \forall x \in \mathbb{R}, p(x) \rightarrow q_1(x) \}$$

$$\Leftrightarrow \exists x \in \mathbb{R}, \sim (p(x) \rightarrow q_1(x))$$

$$\Leftrightarrow \exists x \in \mathbb{R}, \sim (\sim p(x) \vee q_1(x))$$

$$\Leftrightarrow \exists x \in \mathbb{R}, p(x) \wedge \sim q_1(x)$$

In words, For some real no, $|x-3| < 7$ and

$$x \notin (-4, 10).$$

(iv) $p(x): x^2 > 16$, $q(x): x < -4$, $r(x): x > 4$

Symbolic form, $\forall x \in \mathbb{R}, p(x) \rightarrow (q(x) \vee r(x))$

Its negation is

$$\sim \{ \forall x \in \mathbb{R}, [p(x) \rightarrow (q(x) \vee r(x))] \}$$

$$\Leftrightarrow \exists x \in \mathbb{R}, \sim [p(x) \rightarrow (q(x) \vee r(x))]$$

$$\Leftrightarrow \exists x \in \mathbb{R}, \sim [\sim p(x) \vee (q(x) \vee r(x))]$$

$$\Leftrightarrow \exists x \in \mathbb{R}, p(x) \wedge \sim (q(x) \vee r(x))$$

$$\Leftrightarrow \exists x \in \mathbb{R}, p(x) \wedge \sim q(x) \wedge \sim r(x)$$

In words,

For some real x_0 , $x^2 > 16$ and $x \geq -4$ and $x \leq 4$.

(*) Logical Implication Involving Quantifiers

A quantified statement P is said to logically imply a quantified statement Q if Q is true whenever P is true
then we write $P \Rightarrow Q$

An argument $P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n \rightarrow Q$ is valid argument if Q is true whenever each of $P_1, P_2, \dots, P_n$ is true (or)

$$P_1 \wedge P_2 \wedge P_3 \wedge \dots \wedge P_n \Rightarrow Q$$

PROBLEMS

(1) p.t the following argument is valid

All men are mortal

Sachin is a man

$\therefore$ Sachin is mortal

$\therefore$ let S denote the set of all men

$p(x) : x$ is mortal

$a : \text{Sachin}$

The given argument,

$$\begin{array}{c} \forall x \in S, p(x) \\ a \in S \\ \hline \therefore p(a) \end{array}$$

$p(x)$ is true and $a \in S$, means $p(a)$ is true

$\therefore$ This is valid by universal specification.

(2) Find whether the following is a valid argument for which the universe is the set of all students

No Engineering student is bad in studies

Anil is not bad in studies

$\therefore$ Anil is an engineering student.

Soln:- S : set of all students.

$p(x) : x$ is an engineering student

$q(x) : x$ is bad in studies

$a : \text{Anil.}$

Argument,

$$\forall x, [p(x) \rightarrow \sim q(x)]$$

$$\sim q(a)$$

$$\therefore p(a)$$

$$[p(x) \rightarrow \sim q(x)] \wedge \sim q(a)$$

$$\Leftrightarrow [p(a) \rightarrow \sim q(a)] \wedge \sim q(a) \quad (\text{by universal specification})$$

$$\nRightarrow p(a) \quad (\because \text{It is not M.T (or M.P.)})$$

Because no rule is applicable

$\therefore$ It is an invalid statement.

Q3) Find whether the following variable is valid.

No engineering students of Ist or IInd sem studies logic

Anil is an engineering student who studies logic

$\therefore$ Anil is not in second semester.

Soln:- S : set of all engineering students.

$p(x)$: x is in Ist sem

$q(x)$: x is in IInd sem

$r(x)$: x studies logic

a : Anil

$$\therefore \forall x \in S, \frac{(p(x) \vee q(x)) \rightarrow \sim r(x)}{r(a)}$$

$$\therefore \sim q(a)$$

$$\{ (p(x) \vee q(x)) \rightarrow \sim r(x) \} \wedge r(a)$$

$$\Rightarrow \{ (p(a) \vee q(a)) \rightarrow \sim r(a) \} \wedge r(a)$$

$$\Rightarrow \sim [p(a) \vee q(a)] \quad \begin{array}{l} \text{[WKT by MT} \\ (p \rightarrow \sim q) \wedge q \Leftrightarrow \\ \sim p \end{array}$$

$$\Rightarrow \sim p(a) \wedge \sim q(a)$$

by conjunction simplification

$$\Rightarrow \sim q(a) \wedge \sim p(a) \quad \text{commutative}$$

$$\Rightarrow \sim q(a) \quad (\text{by conjunction simplification})$$

It is valid.

^{sup}
(4) p.T the following argument is not valid

All squares have four sides

The quadrilateral ABCD has four sides

$\therefore$ ABCD is a square

Soln:- S : set of all quadrilateral

$p(x)$: x is a square

$q(x)$: x has 4 sides.

a : ABCD

$$\forall x \in S, p(x) \rightarrow q(x)$$

$$\frac{q(a)}{\therefore p(a)}$$

$$[p(x) \rightarrow q(x)] \wedge q(a)$$

$$\Rightarrow [p(a) \rightarrow q(a)] \wedge q(a) \quad [\text{neither M.P nor M.T}]$$

$$\nRightarrow p(a)$$

$\therefore$ It is valid.

✓ (5) Over the universe of all quadrilateral in plane geometry. Verify the validity of the argument "since every square is a rectangle and every rectangle is a parallelogram, it follows that every square is a parallelogram."

soln:- S : set of all ~~para~~ quadrilateral

$p(x)$: x is a square

$q(x)$: x is a rectangle

$r(x)$: x is a parallelogram.

$$\forall x \in S, p(x) \rightarrow q(x)$$

$$q(x) \rightarrow r(x)$$

$$\therefore p(x) \rightarrow r(x)$$

$$\text{Now, } [p(x) \rightarrow q(x)] \wedge [q(x) \rightarrow r(x)]$$

$$\Rightarrow [p(a) \rightarrow q(a)] \vee [q(a) \rightarrow r(a)]$$

$$\Rightarrow p(a) \rightarrow r(a) \quad [\therefore \text{Syllogism}]$$

$\therefore$ It is valid.

^{np} (6) prove the following argument is valid

$$\forall x, [p(x) \rightarrow \{q(x) \vee r(x)\}]$$

$$\forall x, [p(x) \vee s(x)]$$

$$\therefore \forall x, [r(x) \vee s(x)]$$

soln:- $[p(x) \rightarrow \{q(x) \vee r(x)\}] \vee [p(x) \vee s(x)]$

$$\Rightarrow [p(x) \rightarrow \{q(x) \vee r(x)\}] \vee [p(x) \vee s(x)] \quad \xrightarrow{\text{M.P}}$$

$$\Rightarrow \{q(x) \vee r(x)\} \vee s(x)$$

$$\Rightarrow r(x) \vee s(x) \quad (\text{by conjunctive simplification})$$

$$\Rightarrow r(x) \vee s(x)$$

It is valid.

^{np} (7) prove the following argument is valid

$$\forall x, [p(x) \vee q(x)]$$

$$\forall x, [\sim p(x) \vee \{q(x) \vee r(x)\}]$$

$$\therefore \forall x, [\sim p(x) \rightarrow r(x)]$$

soln:- $[p(x) \vee q(x)] \vee [\sim p(x) \vee \{q(x) \vee r(x)\}]$

$$\Rightarrow [p(a) \vee q(a)] \wedge [\sim p(a) \wedge q(a)] \rightarrow r(a) \quad \text{Law for condn.}$$

$$\Rightarrow [p(a) \vee q(a)] \wedge [\sim(\sim p(a) \wedge q(a)) \vee r(a)]$$

demorgan's

$$\Rightarrow [p(a) \vee q(a)] \wedge [p(a) \vee \sim q(a)] \vee r(a)$$

distributive

$$\Rightarrow [p(a) \vee (q(a) \wedge \sim q(a))] \vee r(a)$$

$$\Rightarrow p(a) \vee F_0 \vee r(a)$$

$$\Rightarrow p(a) \vee r(a)$$

$$\Rightarrow r(a) \vee p(a)$$

$$\Rightarrow \sim \sim r(a) \vee p(a)$$

$$\Rightarrow \sim r(a) \rightarrow p(a)$$

$$\Rightarrow \sim r(x) \rightarrow p(x) //$$

It is valid.

(8) prove that the following argument is valid

$$\forall x, [p(x) \vee q(x)]$$

$$\exists x, \sim p(x)$$

$$\forall x, [q(x) \vee \sim r(x)]$$

$$\forall x, [r(x) \rightarrow \sim s(x)]$$

$$\therefore \exists x, \sim s(x)$$

Soln:- $[p(x) \vee q(x)] \wedge \sim p(x) \wedge [q(x) \vee \sim r(x)] \wedge [r(x) \rightarrow \sim s(x)]$

$$\Rightarrow [p(a) \vee q(a)] \wedge \sim p(a) \wedge [q(a) \vee \sim r(a)] \wedge [r(a) \rightarrow \sim s(a)]$$

by disjunctive syllogism

$$\Rightarrow \underline{q(a) \wedge [\sim q(a) \vee r(a)] \wedge [s(a) \rightarrow \sim r(a)]} \quad \begin{array}{l} \text{(by disjunctive} \\ \text{sylllogism)} \end{array}$$

$$\Rightarrow r(a) \wedge [s(a) \rightarrow \sim r(a)]$$

$$(p \vee q) \wedge \sim q \Rightarrow p$$

$$\Rightarrow r(a) \wedge [\sim s(a) \vee \sim r(a)] \quad \text{by disjunctive syllogism}$$

$$\Rightarrow \sim s(a)$$

$$\Rightarrow \sim s(x)$$

$\therefore$ It is valid.

(9) Find whether the following argument is valid.

If a triangle has 2 equal sides, then it is isosceles

If a triangle is isosceles, then it has 2 equal angles

The triangle ABC does not have 2 equal angles

$\therefore$ ABC does not have 2 equal sides.

Soln:- S : set of all triangles.

$p(x)$: x has 2 equal sides

$q(x)$: x is isosceles.

$r(x)$: x has 2 equal angles.

a : $\triangle ABC$ (counter example)

Now, $\forall x \in S, p(x) \rightarrow q(x)$

$q(x) \rightarrow r(x)$

$\sim r(a)$

$\sim p(a)$

$$\therefore [p(x) \rightarrow q(x)] \wedge [q(x) \rightarrow r(x)] \wedge \sim r(a)$$

$$\Rightarrow [p(a) \rightarrow q(a)] \wedge [q(a) \rightarrow r(a)] \wedge \sim r(a)$$

Syllogism

$$\Rightarrow [p(a) \rightarrow r(a)] \wedge \sim r(a)$$

Modus ponens

$$\Rightarrow \sim p(a)$$

$\therefore$ It is valid //

(10) Determine if the argument is valid (or) not.

All people concerned about the environment, recycle their plastic containers. B is not concerned about the environment, therefore, B does not recycle his plastic containers.

Soln:- S : Set of all people.

$p(x)$: x is concerned about the environment

$q(x)$: x is recycle their containers.

a : B

$$\forall x \in S, \quad p(x) \rightarrow q(x)$$

$$\frac{\sim p(a)}{\therefore \sim q(a)}$$

$$\therefore [p(x) \rightarrow q(x)] \wedge \sim p(a) \Rightarrow [p(a) \rightarrow q(a)] \wedge \sim p(a)$$

$\nRightarrow \sim q(a)$ [both MP and MT are invalid.]

$\therefore$ Argument is invalid

(*) Open statements with more than one variable

consider the following statements $x-2y \geq 0$
and $x-y+z=0$, these are the open statements
which contains more than one free variables.

These becomes propositions if each variables is
replaced by an element in a universal set.

Open statements containing two variables are
denoted by $p(x,y)$, $q(x,y)$ -- so on

Open statements containing 3 variables is denoted
by $p(x,y,z)$, $q(x,y,z)$ --- so on

ex: (1) let $p(x,y) : x^2 \geq y$, $q(x,y) : (x+2) < y$, where
 x and y are the set of all real numbers. Determine
the truth values of the following statements.

~~soln:~~ (i) $p(2,4)$ (ii) $p(-3,8)$ (iii) $p(-3,8) \wedge q(1,3)$

(iv) $p(\frac{1}{2}, \frac{1}{3}) \vee \sim q(-2, -3)$

soln: - (i) $p(\underset{x}{2}, \underset{y}{4}) : (2)^2 \geq 4$

$$4 \geq 4 \rightarrow \text{true} - 1$$

(ii) $p(-3,8) : (-3)^2 \geq 8$

$$\Rightarrow 9 \geq 8 \rightarrow \text{true} - 1$$

(iii) $p(-3,8) : -3^2 \geq 8 = 9 \geq 8 - \text{true} - 1$

$$q(1,3): (1+2) < 3$$

$$3 < 3 \rightarrow \text{false} = 0$$

$$\therefore p(-3,8) \wedge q(1,3) = 1 \wedge 0 = 0 \rightarrow \text{False}$$

$$(iv) \quad p\left(\frac{1}{2}, \frac{1}{3}\right): \left(\frac{1}{2}\right)^2 \geq \frac{1}{3}$$

$$\Rightarrow \frac{1}{4} \geq \frac{1}{3} \rightarrow \text{False} = 0$$

$$q(-2,-3): (-2+2) < -3$$

$$\Rightarrow 0 < -3 \Rightarrow \text{false} = 0$$

$$\therefore p\left(\frac{1}{2}, \frac{1}{2}\right) \vee \sim q(-2,-3)$$

$$\Rightarrow 0 \vee 1$$

$$\Rightarrow 1 = \text{true}$$

(*) Quantified Statement with more than one variable

When an open statement contains more than one free variable, quantification may be applied to each of the variable.

properties

$$(i) \quad \forall x, \forall y \quad p(x,y) \Leftrightarrow \forall y, \forall x \quad p(x,y) \Leftrightarrow \forall x,y \quad p(x,y)$$

$$(ii) \quad \exists x, \exists y \quad p(x,y) \Leftrightarrow \exists y, \exists x \quad p(x,y) \Leftrightarrow \exists x,y \quad p(x,y)$$

$$(iii) \quad \forall x, \exists y \quad p(x,y) \not\Leftrightarrow \exists y, \forall x, p(x,y)$$

$$(iv) \quad \forall y, \exists x \quad p(x,y) \not\Leftrightarrow \exists x, \forall y \quad p(x,y)$$

Similarly for 3 variables.

problems

(1) Let x and y denote the integers. Consider the statement

$p(x, y): x+y$ is even

write down the following statements in words

(i) $\forall x, \exists y, p(x, y)$ (ii) $\exists x, \forall y, p(x, y)$

(iii) $\forall x, \forall y, p(x, y)$

Soln:- (i) For All integers x , there exist an integer y , $x+y$ is even

(ii) There exists an integers x such that $x+y$ is even for every integer y .

(iii) For all integers x and y , $x+y$ is even.

(2) Find the Negation for the following ^{quantified} statements

(i) $\forall x, \exists y, [(p(x, y) \vee q(x, y)) \rightarrow r(x, y)]$

Soln:- $\sim \{ \forall x, \exists y, [(p(x, y) \vee q(x, y)) \rightarrow r(x, y)] \}$

$\Rightarrow \exists x, \sim \{ \exists y, [(p(x, y) \vee q(x, y)) \rightarrow r(x, y)] \}$

$\Rightarrow \exists x, \forall y, \sim \{ (p(x, y) \vee q(x, y)) \rightarrow r(x, y) \}$

$\Rightarrow \exists x, \forall y, \sim [\sim (p(x, y) \vee q(x, y)) \vee r(x, y)]$

$\Rightarrow \exists x, \forall y, [p(x, y) \vee q(x, y)] \wedge \sim r(x, y) //$

$$(ii) \forall x, \forall y [(x < y) \rightarrow \exists z, (x < z < y)]$$

$$\text{soln:- } \sim \{ \forall x, \forall y [(x < y) \rightarrow \exists z, (x < z < y)] \}$$

$$\Rightarrow \exists x, \exists y \sim [(x < y) \rightarrow \exists z, (x < z < y)] \quad \text{de Morgan's.}$$

$$\Rightarrow \exists x, \exists y, (x < y) \wedge \sim \exists z, (x < z < y)$$

$$\Rightarrow \exists x, \exists y, (x < y) \wedge \forall z (x > z > y)$$

$$(iii) \forall x, \forall y [|x| = |y| \rightarrow (y = \pm x)]$$

$$\text{soln:- } \sim \{ \forall x, \forall y [|x| = |y| \rightarrow (y = \pm x)] \}$$

$$\Rightarrow \exists x, \exists y \sim [|x| = |y| \rightarrow (y = \pm x)]$$

$$\Rightarrow \exists x, \exists y \sim [\sim(|x| = |y|) \vee (y = \pm x)]$$

$$\Rightarrow \exists x, \exists y (|x| = |y|) \wedge (y \neq \pm x) //$$

$$(iv) [\forall x, \forall y, ((x < 0) \wedge (y > 0))] \rightarrow [\exists z, (xz > y)]$$

soln:-

$$\sim \{ \forall x, \forall y, ((x < 0) \wedge (y > 0)) \rightarrow [\exists z, (xz > y)] \}$$

$$\Rightarrow \sim \{ \sim(\forall x, \forall y, [(x < 0) \wedge (y > 0)]) \vee \exists z, (xz > y) \}$$

$$\Rightarrow \sim \{ \forall x, \forall y, [(x < 0) \wedge (y > 0)] \wedge \sim \exists z, (xz > y) \}$$

$$\Rightarrow \forall x, \forall y, [(x < 0) \wedge (y > 0)] \wedge \forall z, (xz \leq y) //$$

Methods of proof and Disproof

The propositions that commonly appear in mathematical discussions are conditionals of the form $p \rightarrow q$, where p & q are simple or compd proposition & which may involves quantifiers also.

The process of establishing that the conditional is true by using the rules or laws of logic and other known facts constitutes a proof of the conditional.

The process of establishing that a proposition is false constitutes a disproof.

(*) Direct proof

The direct proof of a conditional $p \rightarrow q$ has the following steps.

- (1) Hypothesis : First assume that p is true
- (2) Analysis : Starting with the hypothesis and employing the rules or laws of logic and other known facts, infer that q is true
- (3) Conclusion : $p \rightarrow q$ is true.

problems

(1) Give a direct proof of the statement:

"The square of an odd integer is an odd integer".

Soln:- Is of the form,

"If n is an odd integer, then n^2 is an odd integer"

Now, p : n is an odd integer

q : n^2 is an odd integer.

Hypothesis:- Assume that n is an odd integer

i.e. $n = 2K + 1$ for some ^{integer} K

Analysis: Consequently $n^2 = (2K + 1)^2$

$$n^2 = 4K^2 + 1 + 4K, \text{ which is}$$

not divisible by 2.

$\therefore n^2$ is an odd integer.

Conclusion: $p \rightarrow q$ is true.

^{Q2} (2) p.t, for all integers K and l , if K & l are both odd, then $K+l$ is even and Kl is odd.

Soln:- p : K & l are both odd

q : $K+l$ is even and Kl is odd

Hypo: Assume that K & l are odd

$$\text{i.e. } K = 2m + 1, \quad l = 2n + 1, \quad m, n \in \mathbb{Z}$$

Analy ; consequently ,

$$k+l = (2m+1) + (2n+1)$$

$$k+l = 2m+2n+2, \text{ which is divisible by } 2$$

$\therefore k+l$ is even

$$kl = (2m+1)(2n+1)$$

$$= 4mn + 2m + 2n + 1, \text{ which is not divisible by } 2$$

$\therefore kl$ is odd

concl : $p \rightarrow q$ is true

(*) Indirect proof

WKT $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$ [contrapositive statement]

First show that $\neg q \rightarrow \neg p$ is true by direct proof, this shows that $p \rightarrow q$ is true indirectly, this method of proving a conditional is called indirect proof.

problems

(1) Let n be an integer, p.T "if n^2 is odd then n is odd".

soln:- p : n^2 is odd

q : n is odd

S.T : $\neg q \rightarrow \neg p$ is true

Hypothesis : Assume that $\neg q$ is true

i.e. n is even

$$n = 2K.$$

Analysis : $n^2 = (2K)^2$

$$\Rightarrow n^2 = 4K^2, \text{ divisible by } 2$$

$$\therefore n^2 \text{ is even}$$

Hence $\neg p$ is true

conclusion: $\neg q \rightarrow \neg p$ is true

Hence $p \rightarrow q$ is true.

(2) "The product of two even integers is an even integer".

Soln: If m and n are even integers then mn is an even integer.

p : m & n are even integers

q : mn is an even integer.

S.T : $\neg q \rightarrow \neg p$ is true.

Hypo : Assume that $\neg q$ is true

i.e. mn is an odd integer

Analys:- mn is not divisible by 2

$\Rightarrow m$ is not divisible by 2

n is not divisible by 2

$\therefore m$ is odd & n is odd $\Rightarrow m$ & n are odd integers

$\Rightarrow \sim p$ is true

conclu:- $\sim q \rightarrow \sim p$ is true,

Hence $p \rightarrow q$ is true

(3) For all integers k and l , if $k+l$ is even then
 k & l are both even ($\cap$) both odd

Soln:- p : $k+l$ is even

q : k & l are both even ($\cap$) both odd.

S.T: $\sim q \rightarrow \sim p$ is true

Hypoth: Assume that $\sim q$ is true

i.e k & l are both not even ($\cap$) both not odd
(that means one is even & other one is odd)

i.e $k = 2m$, $l = 2n+1$

Analysis:- $k+l = 2m+2n+1$, is not divisible by 2

$\therefore k+l$ is odd

$\Rightarrow \sim p$ is true

conclu: $\sim q \rightarrow \sim p$ is true

Hence $p \rightarrow q$ is true

(4) provide an indirect proof of the following
Statement. "For all positive real numbers x & y
if the product xy exceeds 25 then $x > 5$ or $y > 5$ ".

Soln: - $p : xy > 25$

S.T: $\neg q \rightarrow \neg p$ is true

$q : x > 5 \text{ or } y > 5$

Hypo: Assume $\neg q$ is true

i.e. $x \leq 5$ and $y \leq 5$

Analysis: $xy \leq 5 \times 5$

$xy \leq 25$

$\Rightarrow \neg p$ is true

conclusion: $\neg q \rightarrow \neg p$ is true

Hence $p \rightarrow q$ is true.

(*) proof by contradiction

Hypothesis: Assume that $p \rightarrow q$ is false

i.e. p is true and q is false.

Analysis: Take q is false, use any rules or laws

(or) known facts to S.T. p is false.

Hence our assumption is wrong.

conclusion: Hence $p \rightarrow q$ is true.

problems:

(1) provide a proof by contradiction of the following statement.

"For every integer n , if n^2 is odd, then n is odd".

Sol:- p : n^2 is odd

q : n is odd

S.T : $p \rightarrow q$ is true by contradiction

Hypothesis : Assume that $p \rightarrow q$ is false

i.e p is true and q is false

Analysis : take q is false

i.e n is even

$$n = 2K$$

consequently, $n^2 = 4K^2$, divisible by 2

$$\Rightarrow n^2 \text{ is even}$$

Hence p is false

Hence our assumption is wrong

conclu : $p \rightarrow q$ is true.

(Q) "If n^2 is an even integer then ' n ' is an even integer".

Soln:- p : n^2 is an even integer

q : n is an even integer.

S.T : $p \rightarrow q$ is true by contradiction

Hypothesis : Assume that $p \rightarrow q$ is false

i.e p is true and q is false

Analysis: take q is false, means n is an odd integer

$$\text{i.e. } n = 2k + 1$$

$$\text{consequently, } n^2 = (2k + 1)^2$$

$$n^2 = 4k^2 + 1 + 4k, \text{ not divisible by } 2$$

$\therefore n^2$ is an odd integer

$\Rightarrow p$ is false

Hence our assumption is wrong

conclu:- $p \rightarrow q$ is true

(3) p.T for all real numbers x and y ,

"if $x + y \geq 100$, then $x \geq 50$ or $y \geq 50$ "

soln:- $p: x + y \geq 100$

$q: x \geq 50$ or $y \geq 50$

Hypo: Assume that $p \rightarrow q$ is false

i.e. p is true and q is false.

Analysis:- take q is false, means $x < 50$ and $y < 50$

$$\text{consequently, } x + y < 50 + 50$$

$$x + y < 100$$

$\Rightarrow p$ is true, hence our assumption is wrong.

conclusion:- $p \rightarrow q$ is true

(A) p.T, "if 'm' is an even integer then 'm+7' is an odd integer".

Soln:- p : m is an even integer
 q : m+7 is an odd integer

Hypo: Assume that $p \rightarrow q$ is false
i.e. p is true and q is false.

Analy: take q is false

m+7 is an even integer

$$\Rightarrow m+7 = 2k$$

$m = 2k + 7$, which is not divisible by 2

$\therefore m$ is an odd integer

$\Rightarrow p$ is false, our assumption is wrong

conclusion: $p \rightarrow q$ is false.

(5) Give a (i) direct proof (ii) indirect proof

(iii) proof by contradiction, for the following statement

"If n is an odd integer, then $n+9$ is an even integer".

Soln:- p : n is an odd integer

q : $n+9$ is an even integer

(i) direct proof:

s.t. $p \rightarrow q$ is true

Hypothesis: Assume that p is true, i.e. n is odd

$$n = 2k + 1$$

Analysis: $n + 9 = 2k + 1 + 9$

$$n + 9 = 2k + 10, \text{ divisible by } 2$$

$\therefore n + 9$ is even integer

$\therefore q$ is true

conclu: $p \rightarrow q$ is true.

(ii) indirect proof:

S.T: $\neg q \rightarrow \neg p$ is true

Hypo: Assume that $\neg q$ is true

i.e. $n + 9$ is an odd integer

Analysis: $n + 9 = 2k + 1$

$$n = 2k + 1 - 9 = 2k - 8, \text{ is divisible by } 2.$$

$\therefore n$ is even integer

i.e. $\neg p$ is true

conclu: $\neg q \rightarrow \neg p$ is true

Hence $p \rightarrow q$ is true

(iii) proof by contradiction:

Hypothesis: Assume $p \rightarrow q$ is false

i.e. p is true and q is false

Analysis: Take q is false

$n + 9$ is an odd integer

$$\therefore n+9 = 2k+1$$

$$n = 2k+1-9$$

$$n = 2k-8, \text{ is divisible by 2}$$

$\therefore n$ is an even integer

i.e. p is false

Hence our assumption is wrong.

conclu:- $p \rightarrow q$ is true.

(*) Disproof by contradiction

Hypothesis: Assume that $p \rightarrow q$ is true

i.e. p is true and q is true

Analysis: Take p is true by using rules, laws or any other known facts to s.t. q is false.

Hence our assumption is wrong.

conclusion: $p \rightarrow q$ is false.

Disproof the Statement:

(1) "If n is an even integer then $n+7$ is an even integer".

soln:- p : n is an even integer

q : $n+7$ is an even integer.

Hypothesis: Assume that $p \rightarrow q$ is true

i.e. p is true and q is true

Analysis: Take p is true, i.e. n is an even integer

$$n = 2k$$

consq, $n+1 = 2k+1$, is not divisible by 2

$\therefore n+1$ is an odd integer

$\therefore q$ is false

Hence our assumption is wrong

conclus: $p \rightarrow q$ is false.

(*) Disproof by counter example:

Disprove the proposition.

(1) The product of any two odd integers is a perfect square.

soln:- take $m=3$ and $n=5$

$$mn = (3)(5) = 15$$

is not a perfect square

Hence the proposition is disproved.