CBCS SCHEME

USN 1VA22CD089

BCS405A

Fourth Semester B.E./B.Tech. Degree Supplementary Examination, June/July 2024

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. M : Marks , L: Bloom's level , C: Course outcomes.

		Module – 1	M	L	C
Q.1	a.	Define Tautology. Show that $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology by constructing the truth table.	6	L1	CO1
	b.	Prove the following using the laws of logic: $P \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$	7	L2	CO1
	c.	Give i) Direct proof ii) indirect proof iii) proof by contradiction for the following statement: "If n is an odd integer then n + 9 is an even integer".	7	L3	CO1
		OR			
Q.2	a.	Test whether the following arguments are valid: $p \rightarrow q$ $r \rightarrow s$ $\underline{\sim q \vee \sim s}$ $\therefore \sim(p \wedge r)$	6	L2	CO1
	b.	Write the following argument in symbolic form and then establish the validity. If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then it has two equal angles. The triangle ABC does not have two equal angles. ∴ ABC does not have two equal sides.	7	L1	CO1
	c.	For the following statements, the universe comprises all non-zero integers. Determine the truth value of each statement: i) $\exists x\, \exists y\, [xy = 1]$ ii) $\exists x\, \forall y\, [xy = 1]$ iii) $\forall x\, \exists y\, [xy = 1]$ iv) $\exists x\, \exists y\, [(2x + y = 5) \wedge (x - 3y = -8)]$ v) $\exists x\, \exists y\, [(3x - y = 7) \wedge (2x + 4y = 3)]$	7	L2	CO1
		Module – 2			
Q.3	a.	Prove that $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \dfrac{n(2n+1)(2n-1)}{3}$ by mathematical Induction.	6	L2	CO2

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	b.	Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and / or 7's.	7	L3	CO2
	c.	Obtain a recursive definition for the sequence $\{a_n\}$ in each of the following cases: i) $a_n = 5n$ ii) $a_n = 3n + 7$ iii) $a_n = 2 - (-1)^n$	7	L3	CO2
		OR			
Q.4	**a.**	Prove that for any positive integer n, $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$, F_n denote the fibonacci number.	6	L2	CO2
	b.	How many arrangement are there for all the letters in the word "SOCIOLOGICAL". In how many of these arrangements. i) A and G are adjacent ii) All vowels are adjacent.	7	L2	CO2
	c.	Determine the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$.	7	L2	CO2
		Module – 3			
Q.5	**a.**	Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by $a^R b$ if and only if "a is a multiple of b". Write down the relation R, relation matrix M(R) and draw its digraph. List out its indegree and out degree.	6	L2	CO3
	b.	Let f and g be functions from R to R defined by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(gof)(x) = 9x^2 - 9x + 3$ determine a and b.	7	L3	CO3
	c.	State Pigeon hole principle. Show that if $n + 1$ numbers are chosen from 1 to $2n$ then atleast one pair add to $2n + 1$.	7	L2	CO3
		OR			
Q.6	**a.**	Let $f : R \rightarrow R$ be defined by $f(x) = \begin{cases} 3x - 5, \text{if } x > 0 \\ 1 - 3x, \text{if } x \leq 0 \end{cases}$ find f(-1), f(5/3), $f^{-1}(0)$, $f^{-1}(-3)$, $f^{-1}([-5, 5])$ and $f^{-1}([-6, 5])$.	6	L1	CO3
	b.	Let f, g, h be functions from Z to Z defined by $f(x) = x - 1$, $g(x) = 3x$, $h(x) = \begin{cases} 0, \text{if } x \text{ is even} \\ 1, \text{if } x \text{ is odd} \end{cases}$ Determine (fo(goh)) (x), ((fog)oh)(x) and verify that fo(goh) = (fog)oh.	7	L2	CO3
	c.	Draw the Hasse (POSET) diagram which represents positive divisors of 36.	7	L2	CO3
		Module – 4			
Q.7	**a.**	In how many ways 5 number of a's, 4 number of b's and 3 number of c's, can be arranged so that all the identical letters are not in a single block.	6	L3	CO4

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<table>
<tr><td></td><td>b.</td><td>Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find that only one chair at each of five tables T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Solve the recurrence relation $a_n = na_{n-1}$ where $n \geq 1$ and $a_0 = 1$.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.8</td><td>a.</td><td>In how many ways can the 26 letters of the English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs?</td><td>6</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>b.</td><td>Find the rook polynomial for the 3 * 3 board by using the expansion formula.
</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td></td><td>c.</td><td>Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ where $n \geq 0$ and $F_0 = 0$, $F_1 = 1$.</td><td>7</td><td>L2</td><td>CO4</td></tr>
<tr><td colspan="6">Module – 5</td></tr>
<tr><td>Q.9</td><td>a.</td><td>Define Group. Show that fourth roots of unity is an abelian group under $\otimes$.</td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b.</td><td>Define Klein 4 group. Verify A = {1, 3, 5, 7} is a Klein 4 group under $\otimes_8$.</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>State and prove Lagrange's theorem.</td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td colspan="6">OR</td></tr>
<tr><td>Q.10</td><td>a.</td><td>If H, K are subgroups of a group G, prove that $H \cap K$ is also a subgroup of G. Is $H \cup K$ a subgroup of G?</td><td>6</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>b.</td><td>Define cyclic group and show that (G, *) whose multiplication table is as given below is cyclic.
<table>
<tr><td>*</td><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr>
<tr><td>a</td><td>a</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td></tr>
<tr><td>b</td><td>b</td><td>c</td><td>d</td><td>e</td><td>f</td><td>a</td></tr>
<tr><td>c</td><td>c</td><td>d</td><td>e</td><td>f</td><td>a</td><td>b</td></tr>
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</table></td><td>7</td><td>L2</td><td>CO5</td></tr>
<tr><td></td><td>c.</td><td>Prove that the only left coset of a subgroup H of a group G which is also a subgroup of G is H itself.</td><td>7</td><td>L2</td><td>CO5</td></tr>
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